

GLOBAL CHAOS SYNCHRONIZATION OF SPROTT-L AND SPROTT-M SYSTEMS BY ACTIVE CONTROL

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ABSTRACT

This paper derives new results for the global chaos synchronization of identical Sprott L systems (1994), identical Sprott M systems (1994) and non-identical Sprott L and M systems. Active control method has been deployed to achieve the global chaos synchronization of the identical and different Sprott L and M systems. Our synchronization results have been established using Lyapunov stability theory. Numerical plots have been presented to show the effectiveness of the active synchronization results derived in this paper for the Sprott L and M systems.

KEYWORDS

Chaos, Chaotic Systems, Synchronization, Active Control, Sprott-L system, Sprott-M system.

1. INTRODUCTION

Chaotic systems are nonlinear systems, which are characterized by the *butterfly effect* [1], viz. high sensitivity to small changes in the initial conditions of the systems. Chaos phenomenon has been extensively studied in the last two decades [1-23]. Chaos theory has been applied in various fields such as Computer Science, Biology, Microbiology, Ecology, Physics, Chemistry, Economics, Secure Communications, Image Processing and Robotics.

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator.

In 1990, Pecora and Carroll [4] devised a novel scheme to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos methods have been applied to various fields, viz. physical systems [5], chemical systems [6], ecological systems [7], secure communications [8-10], etc.

Since the seminal work by Pecora and Carroll [4], various methods have been proposed for the complete chaos synchronization such as OGY method [11], active control method [12-15], adaptive control method [16-20], backstepping method [21-22], sampled-data feedback synchronization method [23], time-delay feedback method [24], sliding mode control method [25-28], etc.

In this paper, new results have been derived for the complete synchronization for identical and different Sprott L and M chaotic systems using active nonlinear control. Using active control and Lyapunov stability theory, we achieve complete synchronization for identical Sprott L systems ([29], 1994), identical Sprott M systems ([29], 1994) and non-identical Sprott L and M systems.

2. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the *master* system described by the dynamics

$$\dot{x} = Ax + f(x) \quad (1)$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \rightarrow R^n$ is the nonlinear part of the system.

As the *slave* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \quad (2)$$

where $y \in R^n$ is the state of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \rightarrow R^n$ is the nonlinear part of the system and $u \in R^n$ is the active controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two different chaotic systems.

In the active control method, we design a feedback controller u , which synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in R^n$.

We define the *complete synchronization error* as

$$e = y - x, \quad (3)$$

From (1), (2) and (3), we obtain the error dynamics as

$$\dot{e} = By - Ax + g(y) - f(x) + u \quad (4)$$

Thus, the complete synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in R^n$.

Hence, we find a feedback controller u so that

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n \quad (5)$$

We take as a candidate Lyapunov function

$$V(e) = e^T P e, \quad (6)$$

where P is a positive definite matrix.

Note that $V : R^n \rightarrow R$ is a positive definite function by construction.

We suppose that the parameters of the master and slave system are known and that the states of both systems (1) and (2) can be measured.

We wish to find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \quad (7)$$

where Q is a positive definite matrix. Then $\dot{V} : R^n \rightarrow R$ is a negative definite function.

Thus, by Lyapunov stability theory [30], the error dynamics (4) is globally exponentially stable.

Hence, it is immediate that the states of the master system (1) and the slave system (2) will be globally and exponentially synchronized.

3. SYSTEMS DESCRIPTION

In this section, we describe the chaotic systems studied in this paper, *viz.* Sprott L and M systems ([29], 1994).

The Sprott-L system is described by the 3D dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 + ax_3 \\ \dot{x}_2 &= bx_1^2 - x_2 \\ \dot{x}_3 &= c - x_1 \end{aligned} \quad (8)$$

where x_1, x_2, x_3 are the states and a, b, c are constant, positive parameters of the system.

The Sprott-L system (8) exhibits a strange chaotic attractor (see Figure 1), when the parameter values are taken as $a = 3.9$, $b = 0.9$ and $c = 1$.

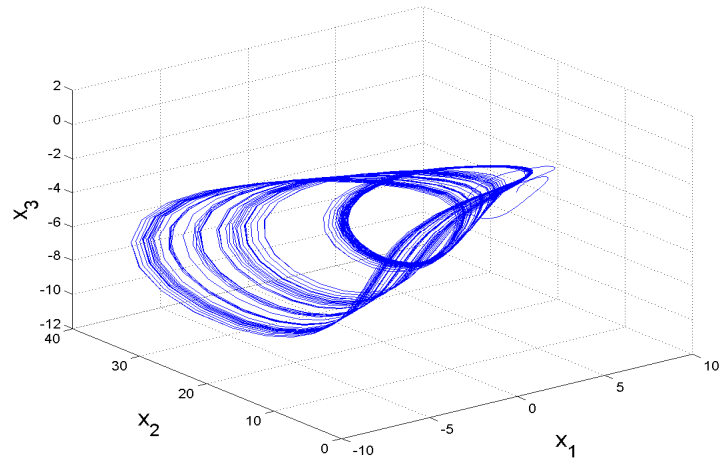


Figure 1. Strange Chaotic Attractor of the Sprott-L System

The Sprott-M system ([29], 1994) is described by the 3D dynamics

$$\begin{aligned} \dot{x}_1 &= -x_3 \\ \dot{x}_2 &= -x_1^2 - \alpha x_2 \\ \dot{x}_3 &= \beta + \gamma x_1 + x_2 \end{aligned} \tag{9}$$

where x_1, x_2, x_3 are the states and α, β, γ are constant, positive parameters of the system.

The Sprott-M dynamics (9) exhibits a chaotic attractor (see Figure 2), when the parameter values are taken as $\alpha = 1, \beta = 1.7$ and $\gamma = 1.7$.

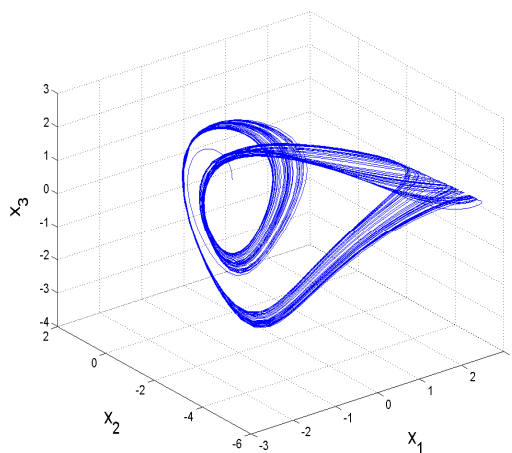


Figure 2. Strange Chaotic Attractor of the Sprott-M System

4. GLOBAL CHAOS SYNCHRONIZATION OF IDENTICAL SPROTT-L SYSTEMS BY ACTIVE CONTROL

4.1 Theoretical Results

In this section, we derive new results for the global chaos synchronization of two identical Sprott-L systems (1994) via the active control method.

Thus, the master system is described by the Sprott-L dynamics

$$\begin{aligned}\dot{x}_1 &= x_2 + ax_3 \\ \dot{x}_2 &= bx_1^2 - x_2 \\ \dot{x}_3 &= c - x_1\end{aligned}\tag{10}$$

where x_1, x_2, x_3 are the state variables and a, b, c are positive parameters of the system.

The slave system is described by the controlled Sprott-L dynamics

$$\begin{aligned}\dot{y}_1 &= y_2 + ay_3 + u_1 \\ \dot{y}_2 &= by_1^2 - y_2 + u_2 \\ \dot{y}_3 &= c - y_1 + u_3\end{aligned}\tag{11}$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the active controls to be designed.

The synchronization error e is defined by

$$\begin{aligned}e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3\end{aligned}\tag{12}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= e_2 + ae_3 + u_1 \\ \dot{e}_2 &= -e_2 + b(y_1^2 - x_1^2) + u_2 \\ \dot{e}_3 &= -e_1 + u_3\end{aligned}\tag{13}$$

We choose the active nonlinear controller as

$$\begin{aligned}u_1 &= -e_2 - ae_3 - k_1 e_1 \\ u_2 &= e_2 - b(y_1^2 - x_1^2) - k_2 e_2 \\ u_3 &= e_1 - k_3 e_3\end{aligned}\tag{14}$$

where the gains $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (14) into (13), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3\end{aligned}\tag{15}$$

Next, we establish the following result.

Theorem4.1. *The identical Sprott-L systems (10) and (11) are globally and exponentially synchronized for all initial conditions with the active controller defined by (12).*

Proof. Consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2),\tag{16}$$

which is a positive definite function on R^3 .

Differentiating (16) along the trajectories of (15), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2\tag{17}$$

which is a negative definite function on R^3 .

Thus, by Lyapunov stability theory [30], the error dynamics (17) is globally exponentially stable.

Hence, it follows that the identical Sprott-L systems (10) and (11) are globally and exponentially synchronized for all initial conditions with the active controller (12).

This completes the proof. ■

4.2 Numerical Results

For simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is deployed to solve the systems (10) and (11) with the active controller (14).

The feedback gains used in the equation (14) are chosen as

$$k_1 = 5, \quad k_2 = 5, \quad k_3 = 5$$

The parameters of the Sprott-L systems are chosen as

$$a = 3.9, \quad b = 0.9, \quad c = 1$$

The initial conditions of the master system (10) are chosen as

$$x_1(0) = 1, \quad x_2(0) = -2, \quad x_3(0) = 2$$

The initial conditions of the slave system (11) are chosen as

$$y_1(0) = 3, \quad y_2(0) = 6, \quad y_3(0) = -2$$

Figure 3 shows the time-history of the synchronization errors e_1, e_2, e_3 .

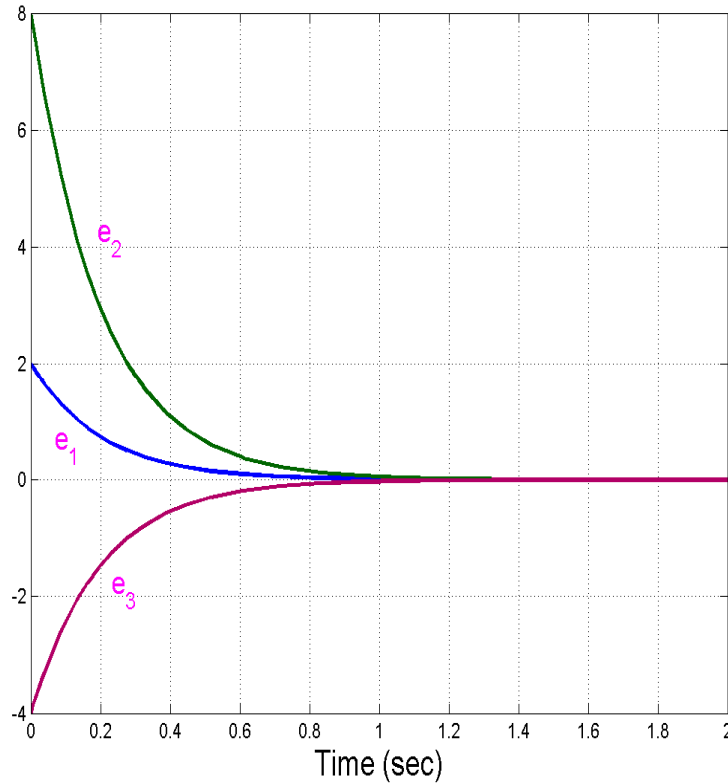


Figure 3. Time-History of the Synchronization Error e_1, e_2, e_3

5. GLOBAL CHAOS SYNCHRONIZATION OF IDENTICAL SPROTT-M SYSTEMS BY ACTIVE CONTROL

5.1 Theoretical Results

In this section, we apply the active nonlinear control method for the synchronization of two identical Sprott-M systems (1994).

Thus, the master system is described by the Sprott-M dynamics

$$\begin{aligned}\dot{x}_1 &= -x_3 \\ \dot{x}_2 &= -x_1^2 - \alpha x_2 \\ \dot{x}_3 &= \beta + \gamma x_1 + x_2\end{aligned}\tag{18}$$

where x_1, x_2, x_3 are the state variables and α, β, γ are positive parameters of the system. The slave system is described by the controlled Sprott-M dynamics

$$\begin{aligned}\dot{y}_1 &= -y_3 + u_1 \\ \dot{y}_2 &= -y_1^2 - \alpha y_2 + u_2 \\ \dot{y}_3 &= \beta + \gamma y_1 + y_2 + u_3\end{aligned}\tag{19}$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the active controls to be designed.

The synchronization error e is defined by

$$\begin{aligned}e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3\end{aligned}\tag{20}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= -e_3 + u_1 \\ \dot{e}_2 &= -\alpha e_2 - y_1^2 + x_1^2 + u_2 \\ \dot{e}_3 &= \gamma e_1 + e_2 + u_3\end{aligned}\tag{21}$$

We choose the active nonlinear controller as

$$\begin{aligned}u_1 &= e_3 - k_1 e_1 \\ u_2 &= \alpha e_2 + y_1^2 - x_1^2 - k_2 e_2 \\ u_3 &= -\gamma e_1 - e_2 - k_3 e_3\end{aligned}\tag{22}$$

where the gains $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (22) into (21), the error dynamics simplifies to

$$\dot{e}_i = -k_i e_i, \quad (i = 1, 2, 3)\tag{23}$$

Theorem 5.1. *The identical Sprott-M systems (18) and (19) are globally and exponentially synchronized for all initial conditions with the active nonlinear controller defined by (22).*

Proof. Consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2),\tag{24}$$

which is a positive definite function on R^3 .

Differentiating (24) along the trajectories of (23), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (25)$$

Clearly, \dot{V} is a negative definite function on R^3 .

Thus, by Lyapunov stability theory [30], the error dynamics (23) is globally exponentially stable.

This completes the proof. ■

5.2 Numerical Results

For simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is deployed to solve the systems (18) and (19) with the active nonlinear controller (22).

The feedback gains used in the equation (22) are chosen as $k_i = 5$, ($i = 1, 2, 3$).

The parameters of the Sprott-M systems are chosen as

$$\alpha = 1, \beta = 1.7, \gamma = 1.7$$

The initial conditions of the master system (20) are chosen as

$$x_1(0) = 2, x_2(0) = 4, x_3(0) = -5$$

The initial conditions of the slave system (21) are chosen as

$$y_1(0) = 6, y_2(0) = -4, y_3(0) = 11$$

Figure 4 shows the time-history of the synchronization errors e_1, e_2, e_3 .

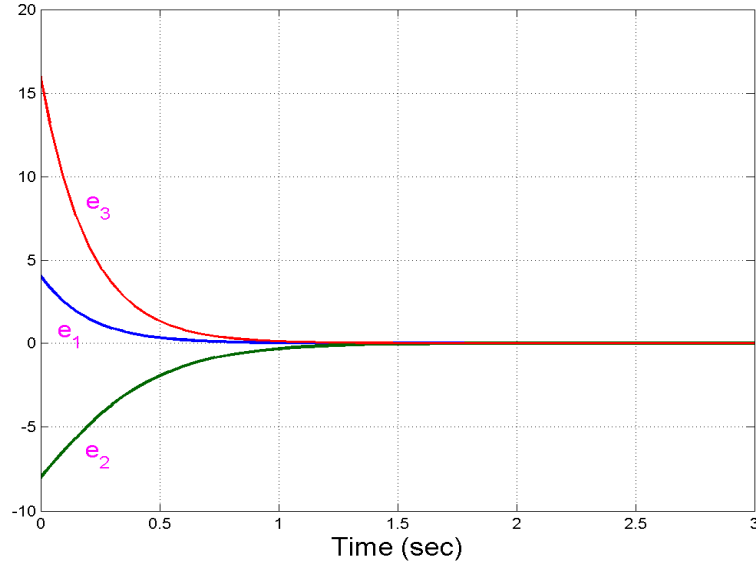


Figure 4. Time-History of the Synchronization Error e_1, e_2, e_3

6. GLOBAL CHAOS SYNCHRONIZATION OF SPROTT-L AND SPROTT-M SYSTEMS BY ACTIVE CONTROL

6.1 Theoretical Results

In this section, we apply the active nonlinear control method for the synchronization of the non-identical Sprott-L and Sprott-M systems (1994).

Thus, the master system is described by the Sprott-L dynamics

$$\begin{aligned}\dot{x}_1 &= x_2 + ax_3 \\ \dot{x}_2 &= bx_1^2 - x_2 \\ \dot{x}_3 &= c - x_1\end{aligned}\tag{26}$$

where x_1, x_2, x_3 are the state variables and a, b, c are positive parameters of the system.

The slave system is described by the controlled Sprott-M dynamics

$$\begin{aligned}\dot{y}_1 &= -y_3 + u_1 \\ \dot{y}_2 &= -y_1^2 - \alpha y_2 + u_2 \\ \dot{y}_3 &= \beta + \gamma y_1 + y_2 + u_3\end{aligned}\tag{27}$$

where y_1, y_2, y_3 are the state variables, α, β, γ are positive parameters and u_1, u_2, u_3 are the active nonlinear controls to be designed.

The synchronization error e is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \quad (28)$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= -y_3 - x_2 - ax_3 + u_1 \\ \dot{e}_2 &= -y_1^2 - bx_1^2 - \alpha y_2 + x_2 + u_2 \\ \dot{e}_3 &= \beta - c + \gamma y_1 + x_1 + y_2 + u_3 \end{aligned} \quad (29)$$

We choose the active nonlinear controller as

$$\begin{aligned} u_1 &= y_3 + x_2 + ax_3 - k_1 e_1 \\ u_2 &= y_1^2 + bx_1^2 + \alpha y_2 - x_2 - k_2 e_2 \\ u_3 &= -\beta + c - \gamma y_1 - x_1 - y_2 - k_3 e_3 \end{aligned} \quad (30)$$

where the gains k_i , ($i = 1, 2, 3$) are positive constants.

Substituting (32) into (31), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \quad (31)$$

Theorem 6.1. *The Sprott-L system (26) and Sprott-M system (27) are globally and exponentially synchronized for all initial conditions with the active controller defined by (30).*

Proof. We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2), \quad (32)$$

which is a positive definite function on R^3 .

Differentiating (32) along the trajectories of (31), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (33)$$

which is a negative definite function on R^3 .

Thus, by Lyapunov stability theory [30], the error dynamics (33) is globally exponentially stable.

This completes the proof. ■

6.2 Numerical Results

For simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ is deployed to solve the systems (26) and (27) with the active nonlinear controller (30).

The feedback gains used in the equation (32) are chosen as $k_i = 5$, ($i = 1, 2, 3$).

The parameters of the Sprott-L systems are chosen as

$$a = 3.9, \quad b = 0.9, \quad c = 1$$

The parameters of the Sprott-M systems are chosen as

$$\alpha = 1, \quad \beta = 1.7, \quad \gamma = 1.7$$

The initial conditions of the master system (26) are chosen as

$$x_1(0) = 2, \quad x_2(0) = 4, \quad x_3(0) = -1$$

The initial conditions of the slave system (27) are chosen as

$$y_1(0) = -2, \quad y_2(0) = 7, \quad y_3(0) = 6$$

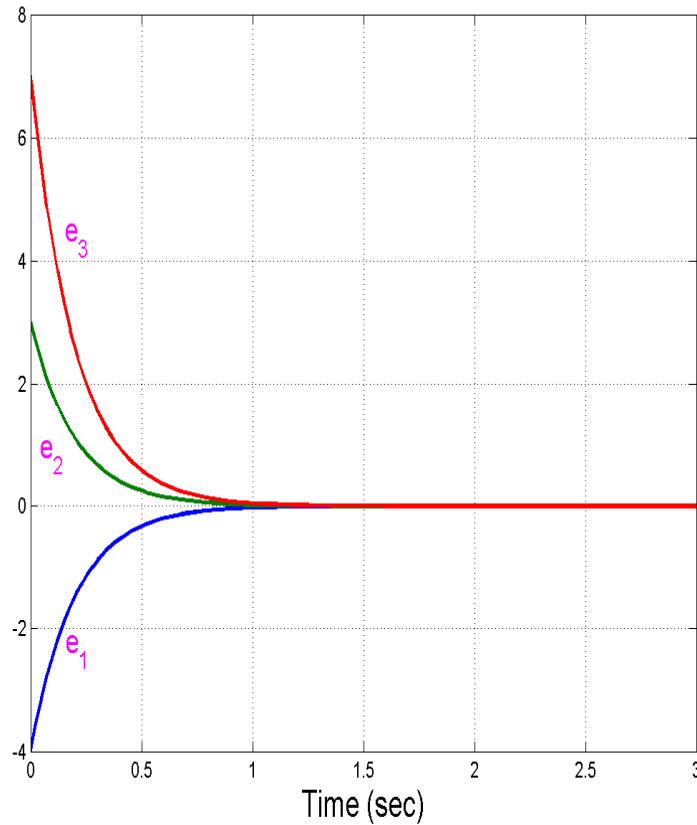


Figure 5. Time-History of the Synchronization Error e_1, e_2, e_3

Figure 5 shows the time-history of the synchronization errors e_1, e_2, e_3 .

7. CONCLUSIONS

In this paper, we derived new results for the global chaos synchronization for the Sprott-L and Sprott-M systems using active control method and we established the synchronization results with the help of Lyapunov stability theory. Numerical simulations have been shown to illustrate the effectiveness of the complete synchronization schemes derived in this paper for the Sprott-L and Sprott-M systems.

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