ADAPTIVE CONTROL AND SYNCHRONIZATION OF HYPERCHAOTIC CAI SYSTEM

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ABSTRACT

The hyperchaotic Cai system (Wang, Cai, Miao and Tian, 2010) is one of the important paradigms of four-dimensional hyperchaotic systems. This paper investigates the adaptive control and synchronization of hyperchaotic Cai system with unknown parameters. First, adaptive control laws are designed to stabilize the hyperchaotic Cai system to its unstable equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then adaptive control laws are derived to achieve global chaos synchronization of identical hyperchaotic Cai systems with unknown parameters. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive control and synchronization schemes.

KEYWORDS

Adaptive Control, Stabilization, Chaos Synchronization, Hyperchaos, Hyperchaotic Cai System.

1. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1]. Since chaos phenomenon in weather models was first observed by Lorenz in 1961, a large number of chaos phenomena and chaos behaviour have been discovered in physical, social, economical, biological and electrical systems.

The control of chaotic systems is to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Active control technique is used when the system parameters are known and adaptive control technique is used when the system parameters are unknown [2-4].

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect, which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem in the chaos literature [5-16].

In 1990, Pecora and Carroll [5] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [6], chemical systems [7], ecological systems [8], secure communications [9-10], etc.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism has been used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Pecora and Carroll [5], a variety of impressive approaches have been proposed for the synchronization of chaotic systems such as the OGY method [11], active control method [12-15], adaptive control method [16-18], sampled-data feedback synchronization method [19], time-delay feedback method [20], backstepping method [21], sliding mode control method [22-24], etc.

Over the last two decades, hyperchaos has been intensively studied in many engineering oriented applied fields such as nonlinear circuits, secure communications, lasers, neural networks and so on.

Hyperchaotic system is usually defined as a chaotic system with at least two positive Lyapunov exponents, implying that its dynamics are expanded in several different directions simultaneously. It means that hyperchaotic systems have more complex dynamical behaviours which can be used to improve the security of a chaotic communication system. Thus, the theoretical design and circuit realization of various hyperchaotic signals have become important research topics [25].

In this paper, we investigate the adaptive control and synchronization of hyperchaotic Cai systems with unknown parameters. First, we devise adaptive stabilization scheme using state feedback control for the hyperchaotic Cai system about its equilibrium at the origin. Then, we devise adaptive synchronization scheme for identical hyperchaotic Cai systems with unknown parameters. The stability results derived in this paper are established using Lyapunov stability theory.

This paper has been organized as follows. In Section 2, we give a system description of the hyperchaotic Cai system (2010). In Section 3, we derive results for the adaptive stabilization of hyperchaotic Cai system with unknown parameters. In Section 4, we derive results for the adaptive synchronization of hyperchaotic Cai systems with unknown parameters. In Section 5, we summarize the main results obtained in this paper.

2. SYSTEM DESCRIPTION

The hyperchaotic Cai system (Wang, Cai, Miao, Tian, [26], 2010) is one of the important models of four-dimensional hyperchaotic systems, which is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 + cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= x_2^2 - rx_3 \\ \dot{x}_4 &= -sx_1 \end{aligned} \tag{1}$$

where x_i $(i = 1, 2, 3, 4)$ are the state variables and a, b, c, r, s are positive constants.

The system (1) is hyperchaotic when the parameter values are taken as

$$a = 27.5, \quad b = 3, \quad c = 19.3, \quad r = 2.9 \text{ and } s = 3.3. \tag{2}$$

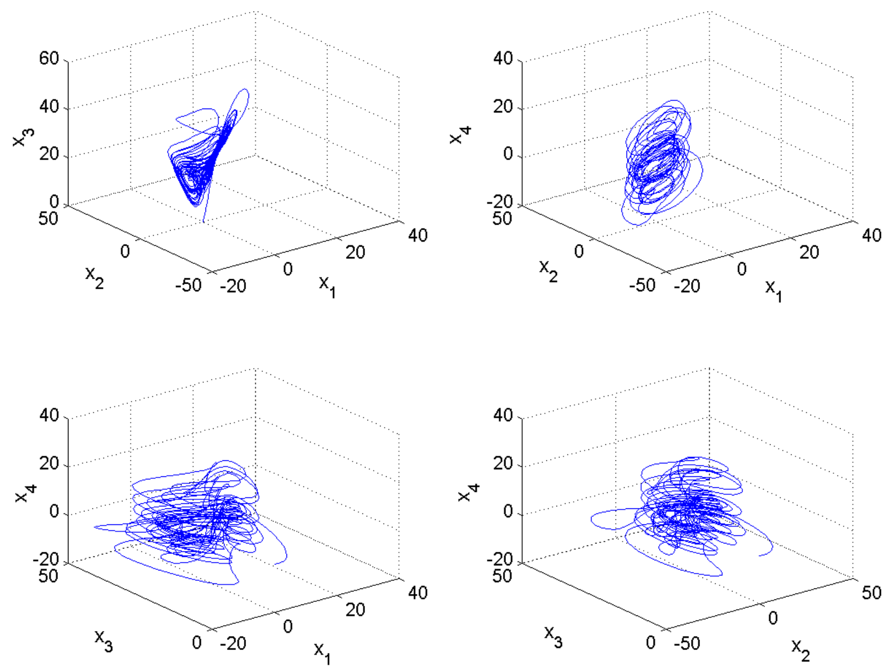

Figure 1. State Orbits of the Hyperchaotic Cai System

The state orbits of the hyperchaotic Cai system (1) are described in Figure 1.

The Lyapunov exponents of the hyperchaotic Cai system (1) can be obtained as

$$\mathrm{LE}_1 = 1.6170, \ \ \mathrm{LE}_2 = 1.6170, \ \ \mathrm{LE}_3 = 0 \ \text{ and } \ \mathrm{LE}_4 = -12.8245. \tag{3}$$

The four-dimensional quadratic autonomous system (1) is hyperchaotic because it has two positive Lyapunov exponents.

When the parameter values are taken as in (2), the system (1) is hyperchaotic and the system linearization matrix at the equilibrium point $E_0 = (0,0,0,0)$ is given by

$$A = \begin{bmatrix} -27.5 & 27.5 & 0 & 0 \\ 3 & 19.3 & 0 & 1 \\ 0 & 0 & -2.9 & 0 \\ -3.3 & 0 & 0 & 0 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = 0.1483, \ \ \lambda_2 = 20.9144, \ \ \lambda_3 = -2.9 \ \text{ and } \ \lambda_4 = -29.2627 \tag{4}$$

Since λ_1, λ_2 are eigenvalues with positive real part, it is immediate from Lyapunov stability theory [27] that the system (1) is unstable at the equilibrium point $E_0 = (0,0,0,0)$.

3. ADAPTIVE CONTROL OF HYPERCHAOTIC CAI SYSTEM

3.1 Theoretical Results

In this section, we design adaptive control law for globally stabilizing the hyperchaotic system (1) when the parameter values are unknown.

Thus, we consider the controlled hyperchaotic Cai system as follows.

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + u_1 \\
\dot{x}_2 &= bx_1 + cx_2 - x_1x_3 + x_4 + u_2 \\
\dot{x}_3 &= x_2^2 - rx_3 + u_3 \\
\dot{x}_4 &= -sx_1 + u_4
\end{aligned} \tag{5}$$

where u_1, u_2, u_3 and u_4 are feedback controllers to be designed using the states and estimates of the unknown parameters of the system.

In order to ensure that the controlled system (5) globally converges to the origin asymptotically, we consider the following adaptive control functions

$$\begin{aligned}
u_1 &= -\hat{a}(x_2 - x_1) - k_1x_1 \\
u_2 &= -\hat{b}x_1 - \hat{c}x_2 + x_1x_3 - x_4 - k_2x_2 \\
u_3 &= -x_2^2 + \hat{r}x_3 - k_3x_3 \\
u_4 &= -\hat{s}x_1 - k_4x_4
\end{aligned} \tag{6}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{r}$ and $\hat{s}$ are estimates of the parameters a, b, c, r and s, respectively, and $k_i, (i = 1,2,3,4)$ are positive constants.

Substituting the control law (6) into the hyperchaotic Cai dynamics (5), we obtain

$$\begin{aligned}
\dot{x}_1 &= (a - \hat{a})\ (x_2 - x_1) - k_1\ x_1 \\
\dot{x}_2 &= (b - \hat{b})x_1 + (c - \hat{c})x_2 - k_2x_2 \\
\dot{x}_3 &= -(r - \hat{r})x_3 - k_3x_3 \\
\dot{x}_4 &= -(s - \hat{s})x_1 - k_4x_4
\end{aligned} \tag{7}$$

Let us now define the parameter errors as

$$\begin{aligned}
&e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c} \\
&e_r = r - \hat{r}, \quad e_s = s - \hat{s}
\end{aligned} \tag{8}$$

Using (8), the closed-loop dynamics (7) can be written compactly as

$$\begin{aligned}\dot{x}_1 &= e_a(x_2 - x_1) - k_1\ x_1 \\ \dot{x}_2 &= e_b x_1 + e_c x_2 - k_2 x_2 \\ \dot{x}_3 &= -e_r x_3 - k_3 x_3 \\ \dot{x}_4 &= -e_s x_1 - k_4 x_4\end{aligned} \tag{9}$$

For the derivation of the update law for adjusting the parameter estimates $\hat{a}, \hat{b}, \hat{c}, \hat{r}, \hat{s},$ the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2 + e_b^2 + e_c^2 + e_r^2 + e_s^2\right), \tag{10}$$

which is a positive definite function on R^9.

Note also that

$$\dot{e}_a = -\dot{\hat{a}},\ \ \dot{e}_b = -\dot{\hat{b}},\ \ \dot{e}_c = -\dot{\hat{c}},\ \ \dot{e}_r = -\dot{\hat{r}},\ \ \dot{e}_s = -\dot{\hat{s}}. \tag{11}$$

Differentiating V along the trajectories of (9) and using (11), we obtain

$$\begin{aligned}\dot{V} &= -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] \\ &\quad + e_b\left[x_1 x_2 - \dot{\hat{b}}\right] + e_c\left[x_2^2 - \dot{\hat{c}}\right] + e_r\left[-x_3^2 - \dot{\hat{r}}\right] + e_s\left[-x_1 x_4 - \dot{\hat{s}}\right]\end{aligned} \tag{12}$$

In view of Eq. (12), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{a}} &= x_1(x_2 - x_1) + k_5 e_a \\ \dot{\hat{b}} &= x_1 x_2 + k_6 e_b \\ \dot{\hat{c}} &= x_2^2 + k_7 e_c \\ \dot{\hat{r}} &= -x_3^2 + k_8 e_r \\ \dot{\hat{s}} &= -x_1 x_4 + k_9 e_s\end{aligned} \tag{13}$$

where $k_i, (i = 5, \ldots, 9)$ are positive constants.

Substituting (13) into (12), we get

$$\dot{V} = -k_1\ x_1^2 - k_2\ x_2^2 - k_3\ x_3^2 - k_4\ x_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_r^2 - k_9 e_s^2 \tag{14}$$

which is a negative definite function on R^9.

Thus, by Lyapunov stability theory [27], we obtain the following result.

Theorem 1. *The hyperchaotic Cai system (5) with unknown parameters is globally and exponentially stabilized for all initial conditions* $x(0) \in R^4$ *by the adaptive control law (6), where the update law for the parameters is given by (13) and* k_i, $(i = 1, \ldots, 10)$ *are positive constants.* ■

2.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the hyperchaotic system (5) with the adaptive control law (6) and the parameter update law (13).

The parameters of the hyperchaotic Cai system (5) are selected as

$$a = 27.5, \quad b = 3, \quad c = 19.3, \quad r = 2.9 \text{ and } s = 3.3.$$

For the adaptive and update laws, we take $k_i = 2, \quad (i = 1, 2, \ldots, 9)$.

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 8, \quad \hat{b}(0) = 9, \quad \hat{c}(0) = 5, \quad \hat{r}(0) = 7 \text{ and } \hat{s}(0) = 10.$$

The initial values of the hyperchaotic Cai system (5) are taken as $x(0) = (12, 8, 20, 32)$.

When the adaptive control law (6) and the parameter update law (13) are used, the controlled hyperchaotic Cai system converges to the equilibrium $E_0 = (0,0,0,0)$ exponentially as shown in Figure 2. The parameter estimates are shown in Figure 3.

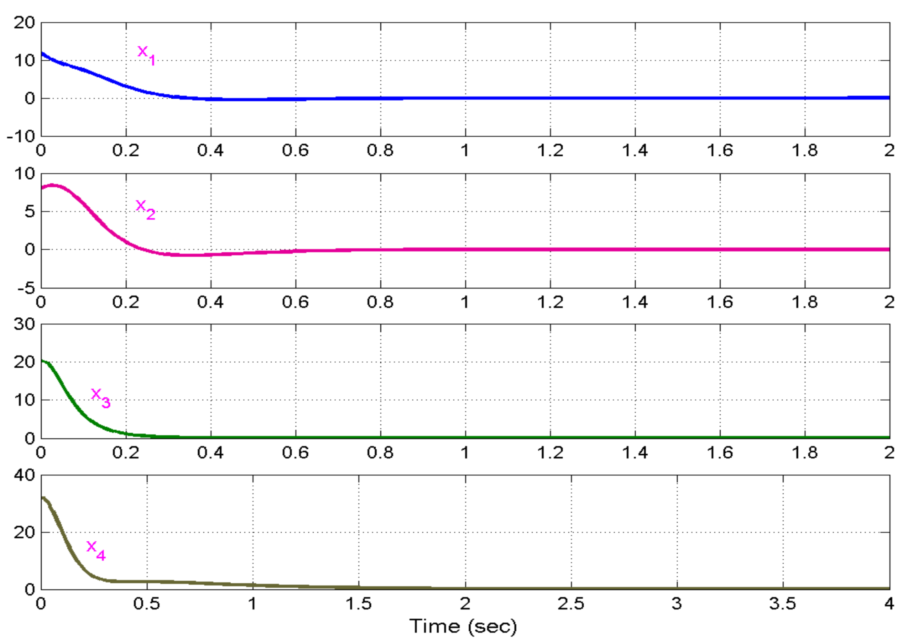

Figure 2. Time Responses of the Controlled Hyperchaotic Cai System

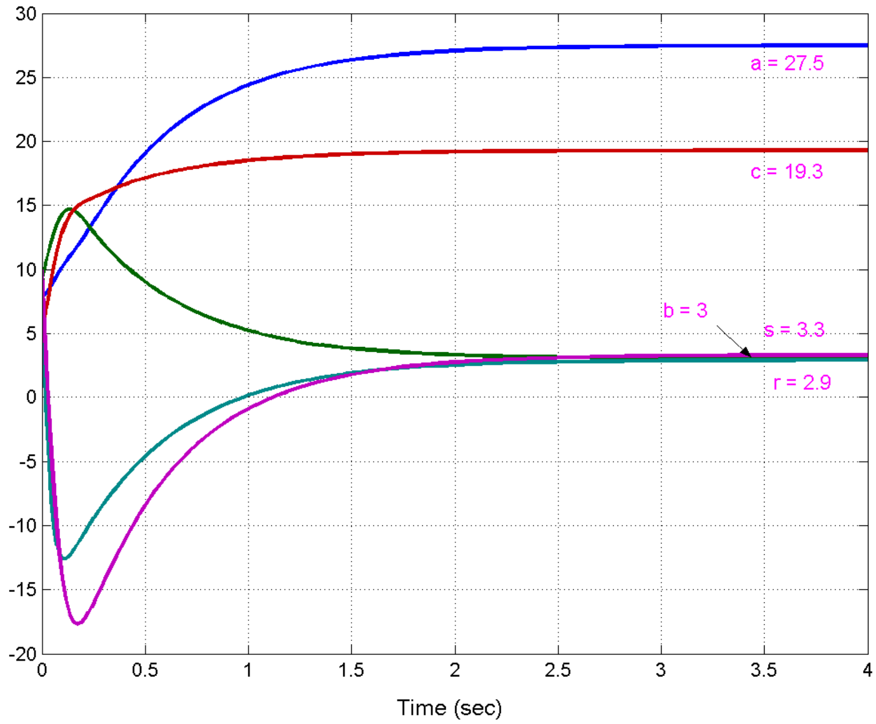

Figure 3. Parameter Estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{r}(t),\ \hat{s}(t)$

4. ADAPTIVE SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC CAI SYSTEMS

4.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical hyperchaotic Cai systems with unknown parameters.

As the master system, we consider the hyperchaotic Cai dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 + cx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= x_2^2 - rx_3 \\ \dot{x}_4 &= -sx_1 \end{aligned} \tag{15}$$

where $x_i,\ (i = 1,2,3,4)$ are the state variables and a,b,c,r,s are unknown system parameters.

The system (15) is hyperchaotic when the parameter values are taken as

$$a = 27.5,\quad b = 3,\quad c = 19.3,\quad r = 2.9 \text{ and } \quad s = 3.3.$$

As the slave system, we consider the controlled hyperchaotic Cai dynamics described by

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= by_1 + cy_2 - y_1 y_3 + y_4 + u_2 \\ \dot{y}_3 &= y_2^2 - ry_3 + u_3 \\ \dot{y}_4 &= -sy_1 + u_4\end{aligned} \tag{16}$$

where y_i, $(i = 1, 2, 3, 4)$ are the state variables and u_i, $(i = 1, 2, 3, 4)$ are the nonlinear controllers to be designed.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \tag{17}$$

Then the error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= be_1 + ce_2 + e_4 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -re_3 + y_2^2 - x_2^2 + u_3 \\ \dot{e}_4 &= -se_1 + u_4\end{aligned} \tag{18}$$

Let us now define the adaptive control functions $u_1(t), u_2(t), u_3(t), u_4(t)$ as

$$\begin{aligned}u_1 &= -\hat{a}(e_2 - e_1) - k_1 e_1 \\ u_2 &= -\hat{b}e_1 - \hat{c}e_2 - e_4 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 &= \hat{r}e_3 - y_2^2 + x_2^2 - k_3 e_3 \\ u_4 &= \hat{s}e_1 - k_4 e_4\end{aligned} \tag{19}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{r}$ and $\hat{s}$ are estimates of the parameters a, b, c, r and s, respectively, and k_1, k_2, k_3, k_4 are positive constants.

Substituting the control law (19) into (18), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= (a - \hat{a})\ (e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= (b - \hat{b})\ e_1 + (c - \hat{c})e_2 - k_2 e_2 \\ \dot{e}_3 &= -(r - \hat{r})\ e_3 - k_3 e_3 \\ \dot{e}_4 &= -(s - \hat{s})e_1 - k_4 e_4\end{aligned} \tag{20}$$

Let us now define the parameter errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c}, \quad e_r = r - \hat{r}, \quad e_s = s - \hat{s} \tag{21}$$

Substituting (21) into (20), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_b e_1 + e_c e_2 - k_2 e_2 \\ \dot{e}_3 &= -e_r e_3 - k_3 e_3 \\ \dot{e}_4 &= -e_s e_1 - k_4 e_4 \end{aligned} \tag{22}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_r^2 + e_s^2\right) \tag{23}$$

which is a positive definite function on R^9.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}}, \quad \dot{e}_r = -\dot{\hat{r}}, \quad \dot{e}_s = -\dot{\hat{s}} \tag{24}$$

Differentiating V along the trajectories of (22) and using (24), we obtain

$$\begin{aligned} \dot{V} = &-k_1\, e_1^2 - k_2\, e_2^2 - k_3\, e_3^2 - k_4\, e_4^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_b\,\left[e_1 e_2 - \dot{\hat{b}}\right] \\ &+ e_c\left[e_2^2 - \dot{\hat{c}}\right] + e_r\left[-e_3^2 - \dot{\hat{r}}\right] + e_s\left[-e_1 e_4 - \dot{\hat{s}}\right] \end{aligned} \tag{25}$$

In view of Eq. (25), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1) + k_5 e_a \\ \dot{\hat{b}} &= e_1 e_2 + k_6 e_b \\ \dot{\hat{c}} &= e_2^2 + k_7 e_c \\ \dot{\hat{r}} &= -e_3^2 + k_8 e_r \\ \dot{\hat{s}} &= -e_1 e_4 + k_9 e_s \end{aligned} \tag{26}$$

where $k_i, (i = 5, \ldots, 9)$ are positive constants.

Substituting (24) into (23), we get

$$\dot{V} = -k_1\, e_1^2 - k_2\, e_2^2 - k_3\, e_3^2 - k_4\, e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_r^2 - k_9 e_s^2, \tag{27}$$

which is a negative definite function on R^9.

Thus, by Lyapunov stability theory [27], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions.

Hence, we have proved the following result.

Theorem 2. *The identical hyperchaotic Cai systems (15) and (16) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (19), where the update law for parameters is given by (26) and* $k_i, (i = 1,\ldots,9)$ *are positive constants.* ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (15) and (16) with the adaptive control law (19) and the parameter update law (26).

For the adaptive synchronization of the hyperchaotic Cai systems with parameter values

$$a = 27.5, \quad b = 3, \quad c = 19.3, \quad r = 2.9 \text{ and } s = 3.3,$$

we apply the adaptive control law (19) and the parameter update law (26).

We take the positive constants $k_i, \ (i = 1,\ldots,9)$ as

$$k_i = 2 \text{ for } i = 1,2,\ldots,9.$$

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 8, \quad \hat{b}(0) = 12, \quad \hat{c}(0) = 9, \quad \hat{r}(0) = 6 \text{ and } \hat{s}(0) = 7.$$

We take the initial values of the master system (13) as

$$x_1(0) = 12, \quad x_2(0) = 20, \quad x_3(0) = 14, \quad x_4(0) = 18$$

We take the initial values of the slave system (14) as

$$y_1(0) = 25, \quad y_2(0) = 30, \quad y_3(0) = 22, \quad y_4(0) = 28$$

Figure 4 shows the adaptive chaos synchronization of the identical hyperchaotic Cai systems. Figure 5 shows that the estimated values of the parameters $\hat{a}, \hat{b}, \hat{c}, \hat{r}, \hat{s}$ converge to the system parameters $a = 27.5, \ b = 3, \ c = 19.3, \ r = 2.9$ and $s = 3.3$.

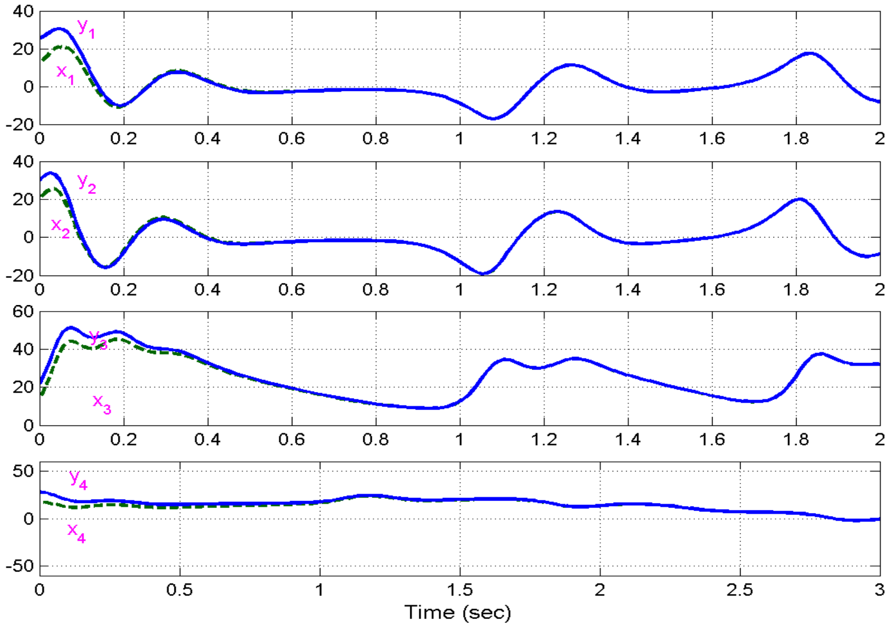

Figure 4. Adaptive Synchronization of the Identical Hyperchaotic Cai Systems

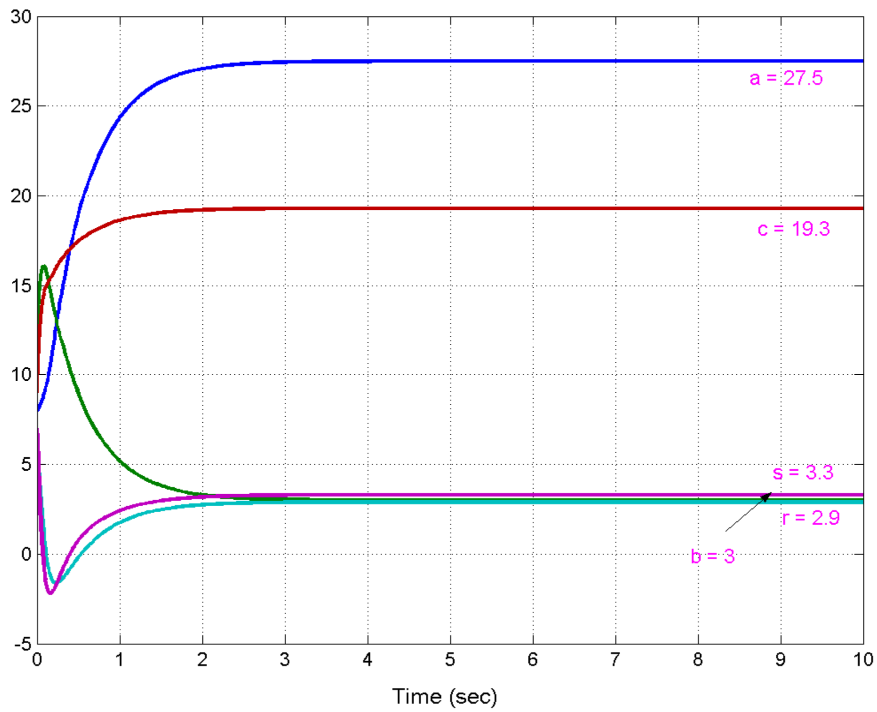

Figure 5. Parameter Estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{r}(t),\ \hat{s}(t)$

5. CONCLUSIONS

In this paper, we applied adaptive control theory for the stabilization and synchronization of the hyperchaotic Cai system (Wang, Cai, Miao, Tian, [26], 2010) with unknown system parameters. First, we designed adaptive control laws to stabilize the hyperchaotic Cai system to its equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then we derived adaptive synchronization scheme and update law for the estimation of system parameters for identical hyperchaotic Cai systems with unknown parameters. Our synchronization schemes were established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient to achieve chaos control and synchronization of the hyperchaotic Cai system. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive stabilization and synchronization schemes.

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