

# MIMO DOWNLINK PRECODING WITH CHANNEL MISMATCH ERROR FOR SIMPLIFIED RECEIVER

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## ABSTRACT

Recently research has been done on linear precoder design to minimize the receiver complexity. In our work, we design the linear precoder for the multiple input multiple output (MIMO) wireless communication system with multiple antennas at the base station (BS) and users each with multiple receiver antennas. Previous works on linear precoder design assume that perfect Channel State Info(CSI) is available at the Base station. But channel estimation error is unavoidable, due to the presence of noise in the channel estimation and due to feedback quantization. In a system employing Time Division Duplexing (TDD), CSI can be obtained at the base station if there is reciprocity between the forward and reverse channels. Channel estimation errors occur due to the presence of background noise in the estimated signal. We derive an MMSE based precoding technique for a MIMO system that considers channel estimation errors as an integral part of the system design. The proposed precoding technique significantly improves the average bit error rate (BER).

## KEYWORDS

MIMO antenna systems, Downlink Precoding, MMSE, transmit optimization, transmit single processing.

## 1. INTRODUCTION

Multiple input Multiple output (MIMO) communication systems play a key role in future wireless communications, because MIMO channels can provide improvement in data rate and reliability. [1],[2],[3],[4],[5] An intensive research is the study of multi-user (MU) MIMO systems. The major limitations in MUMIMO systems are caused interference and channel fading. There can be mitigated by precoding the signals before transmission which requires the knowledge of CSI at the Base Station (BS). Information theoretical analysis in [3],[6],[7],[8],[9] have shown that the capacity of a broadcast MU-MIMO channel can be achieved by applying Dirty-Paper Coding (DPC) as a precoder (or) Tomlinson-Harashima precoder. However these techniques are hard to implement in practice. There are lower complexity linear precoding techniques such as channel inversion [10], regularised channel inversion [11]-[12] and minimum mean squared error (MMSE) precoding [13],[14]. All the above non linear and linear precoding schemes assume full CSI available at the base station. In Time Division Duplexing (TDD) systems [15] CSI can be obtained at the Base Station by exploiting reciprocity between the forward and reverse links. In frequency division duplexing (FDD) system, CSI can be obtained through feedback. But the CSI obtained at the BS is imperfect. The impact of channel estimation errors on the performance of MIMO communication systems is analyzed in [17]-[23].

In [24], D.J. Love et.al designed the precoding techniques, for the downlink of a Multi User Wireless communication system with multiple antennas at BS and users each with a single receive antenna that considers channel estimation error as an integral part of the system design. He also

showed when channel mismatch occurs, his proposed techniques outperformed the previous precoding techniques such as channel inversion and regularised channel inversion. The MMSE precoder with channel estimation error for a MISO TDD-CDMA system has been designed and analyzed in [25].

In our work we extend this result to MIMO wireless systems for simplified receivers. Previous work on transmit pre-processing techniques for MIMO wireless communication systems with the simplified receivers assumes the full CSI available at the BS [13]. Hence, in our work, we derive the transmit precoder for a MIMO wireless communication system with the simplified receiver structure by considering the channel estimation errors. In this paper,  $\text{Tr}()$ ,  $(\cdot)^*$ ,  $(\cdot)^T$  are denoted for the trace of a matrix, Hermitian of a matrix and transpose of a matrix respectively

This paper is organised as follows. In section 2, we discuss the system model for the MIMO wireless communication system for a simplified receiver and section 3 deals with the design of precoding matrix by considering the channel estimation error as an integral part of the design for the system mentioned in the section 2. The simulation results are discussed in section 4 and section 5 concludes this paper.

## 2. SYSTEM MODEL

In our system we consider the transmitter with M transmits antennas and N pairs of Receive antennas. Only one data stream is received by the multiple receive antennas to achieve receive antenna diversity. To attain simple receive diversity scheme [13], the reverse of the simple transmit scheme, which has been proposed by Alamoti in [26], is applied. In this simple receive diversity scheme, we can achieve a diversity order of NM.

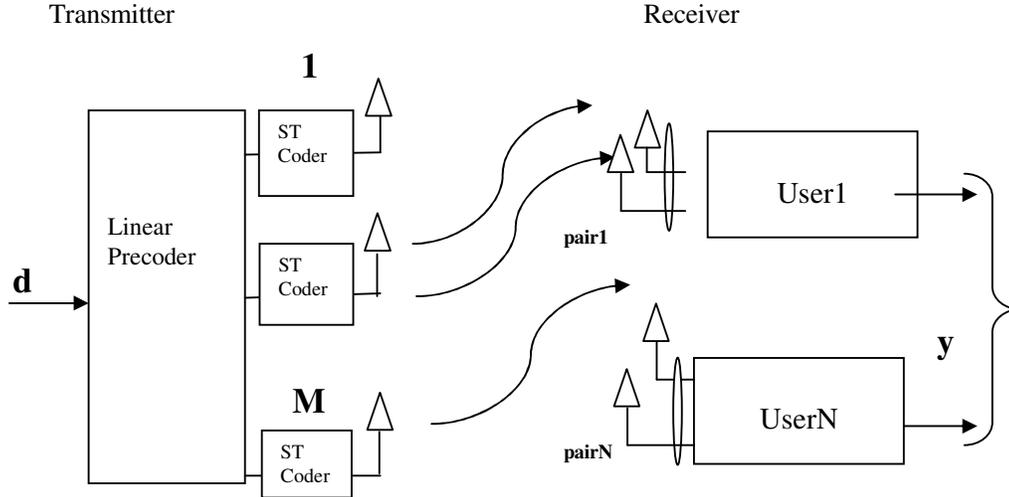


Figure 1. System Model

Let  $\mathbf{d}$  represents the  $N \times 1$  transmit symbol vector and is given by  $\mathbf{d} = [\mathbf{d}_1 \mathbf{d}_2 \dots \mathbf{d}_N]^T$  with  $\mathbf{d}_n$  denoting the two successive data symbols intended for the antenna pair n,  $\mathbf{d}_n = [d_n(2i) \ d_n(2i+1)]^T$ . The overall precoding matrix for the data symbols is  $H_n^*$  and is given by

$$H_n^* = [H_n^{(1)} \ H_n^{(2)} \ \dots \ H_n^{(M)}]^* \quad (1)$$

where  $H_n^{(m)}$  is the precoding matrix for the transmit antenna m and is given by

$$H_n^{(m)} = \begin{pmatrix} h_{n,1}^{(m)} & h_{n,2}^{(m)*} \\ h_{n,2}^{(m)} & -h_{n,1}^{(m)*} \end{pmatrix} \quad (2)$$

with  $h_{n,1}^{(m)}$  and  $h_{n,2}^{(m)}$  denoting the channel from the transmit antenna m to receive antenna 1 and receive antenna 2 of the antenna pair n, respectively. Then, the decision statistics of the successive two symbols at the N pairs of receive antennas in a vector form as  $\mathbf{y} = [y_1 y_2 \dots y_N]^t$  and is given by

$$\mathbf{y} = aHH^* T \mathbf{d} + \mathbf{v} \quad (3)$$

where  $\mathbf{v} = [v_1 v_2 \dots v_N]^t$ ,  $\mathbf{v}_n$  is an additive white Gaussian noise vector  $CN(\mathbf{0}, \mathbf{N}_0 \mathbf{I})$  T is the transmit pre-processing matrix, which is done before the transmit precoding for the simple receive diversity and a is the receiver gain which is common to all receive data stream.

Now consider the situation where the channel  $H$  is imperfectly known to the receiver, due to the channel estimation errors, reciprocity mismatch quantization or delay and is denoted as  $\hat{H}$ . We assume that the channel  $H$  and the channel estimate  $\hat{H}$  are jointly ergodic and stationary Gaussian process and the entries of  $\hat{H}$  are independent. We assume that the estimation error matrix  $\tilde{E} = H - \hat{H}$  has independent elements with zero mean and estimated error variance denoted by  $\sigma_e^2$  and is known to both the transmitter and the receiver. Also, we assume that  $\tilde{E}$  is independent of the data vector  $\mathbf{d}$ ,  $\hat{H}$  and the noise vector  $\mathbf{v}$ .

In our work, the transmitter pre-processing matrix,  $T_{opt}$ , is designed based on the knowledge of the estimated channel matrix  $\hat{H}$ . The transmitter pre-processing matrix,  $T_{opt}$  is designed to minimize the Mean Square Error (MSE) signal at the different users' receivers. The optimization criterion to minimize the MSE is as follows

$$T_{opt} = \underset{\text{Tr}(T^*T) \leq P}{\text{argmin}} E [ \| aHH^* T \mathbf{d} + a\mathbf{v} - \mathbf{d} \|_2^2 | \hat{H} ] \quad (4)$$

where  $\| \cdot \|_2$  denotes the vector Euclidean norm and  $\text{Tr}(T^*T) \leq P$  is the power constraint which states that the total average transmit power after pre-processing by T is less than or equal to P

### 3. TRANSMIT PRE-PROCESSING WITH CHANNEL ESTIMATION ERROR

In this section we derive the optimal pre-processing matrix to minimize the MSE objective function given the estimated channel matrix  $\hat{H}$  at the base station. In [27], [28], and [29], the power constrained MMSE optimization problem was considered without the channel estimation errors. In [24], the above optimization problem is considered with channel estimation error as an integral part of the system design for the MISO system. In our work, we consider the same for the MIMO system with simplified receiver structure. The optimization criterion is again written as

$$T_{opt} = \underset{\text{Tr}(T^*T) \leq P}{\text{argmin}} E [ \| aHH^* T \mathbf{d} + a\mathbf{v} - \mathbf{d} \|_2^2 | \hat{H} ] \quad (5)$$

Since the noise vector  $\mathbf{v}$  is independent of the data vector  $\mathbf{d}$  and the channel matrix  $H$ , the objective function is written as

$$E [ \| aHH^* T \mathbf{d} + a\mathbf{v} - \mathbf{d} \|_2^2 | \hat{H} ] = E [ \| aHH^* T \mathbf{d} - \mathbf{d} \|_2^2 | \hat{H} ] + a^2 E[\mathbf{v}^* \mathbf{v}]$$

If we let  $S = aT$ , and we assume that  $\text{Tr}(T^*T) = P$ , then the optimal pre-processing matrix  $T_{\text{opt}}$  is obtained from

$$S_{\text{opt}} = \underset{\text{Tr}(S^*S) \leq a^2P}{\text{argmin}} E [ \| HH^* S \mathbf{d} - \mathbf{d} \|_2^2 | \hat{H} ] + \text{Tr}(S^*S) KN_0 / P \quad (6)$$

Let us take the first term in the above equation

$$E [ \| HH^* S \mathbf{d} - \mathbf{d} \|_2^2 | \hat{H} ] = E [ \mathbf{d}^*(HH^* S - \mathbf{I})^* (HH^* S - \mathbf{I}) \mathbf{d} | \hat{H} ] \quad (7)$$

If we let  $H = \hat{H} + \check{E}$ , and also we assume that the channel estimation error  $\check{E}$  is independent of the data vector  $\mathbf{d}$ , we get the simplified form of the above equation as

$$= \text{Tr} ( S^* \hat{H}^* \hat{H} \hat{H}^* S ) - 2 \text{Re} ( \text{Tr} ( \hat{H} \hat{H}^* S ) ) + 3K \sigma_e^4 \text{Tr} ( S^* S ) + C \quad (8)$$

where  $3\sigma_e^4$  is the fourth order central moment.

To find the optimal pre-processing matrix  $S$  the gradient of the above equation has to be found as follows,

$$\nabla_S ( \text{Tr} ( \text{Tr} ( S^* \hat{H}^* \hat{H} \hat{H}^* S ) - 2 \text{Re} ( \text{Tr} ( \hat{H} \hat{H}^* S ) ) + 3K \sigma_e^4 \text{Tr} ( S^* S ) + C + \text{Tr}(S^*S) KN_0/P ) = S^* \hat{H} \hat{H}^* \hat{H} \hat{H}^* - \hat{H} \hat{H}^* + 3 K \sigma_e^4 S^* + KN_0 S^*/P = 0 \quad (9)$$

$$S_{\text{opt}} = \hat{H} \hat{H}^* ( 3 K \sigma_e^4 + KN_0/P \hat{H} \hat{H}^* \hat{H} \hat{H}^* )^{-1} \quad (10)$$

## 4. SIMULATION RESULTS AND DISCUSSIONS

In the following, we discuss the simulation results of the system described in section 2 and 3. Here, we assume that the channel is flat fading with Rayleigh distribution. We also assume that the channel estimation errors are generated from an independent Gaussian process with known

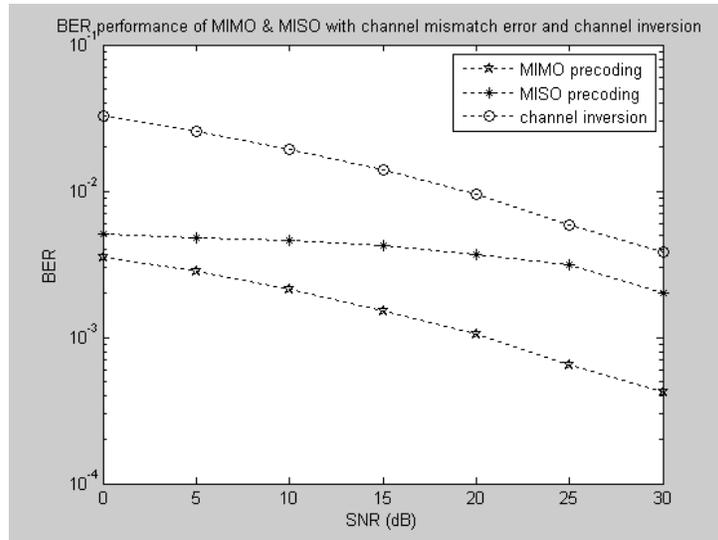


Figure2. Performance of MMSE precoding of MISO and MIMO system for  $M=2, N=2, K=2$  and  $\sigma_e^2 = .1$

variance. Here, we assume that the channel is flat fading with Rayleigh distribution. We also assume that the channel estimation errors are generated from an independent Gaussian process with known variance. In the first simulation, we compare the proposed system with the system described in [24] and the channel inversion precoding system. In the above simulation, we consider a system with two transmit antennas at the base station and two users each with single receive antenna for the MISO system proposed by [24], and two users with two receive antennas each for our proposed MIMO simple receive diversity scheme. The channel estimation error variance is 0.1. The BER performance of our proposed system outperforms the MISO system and the channel inversion system for all the values of SNR.

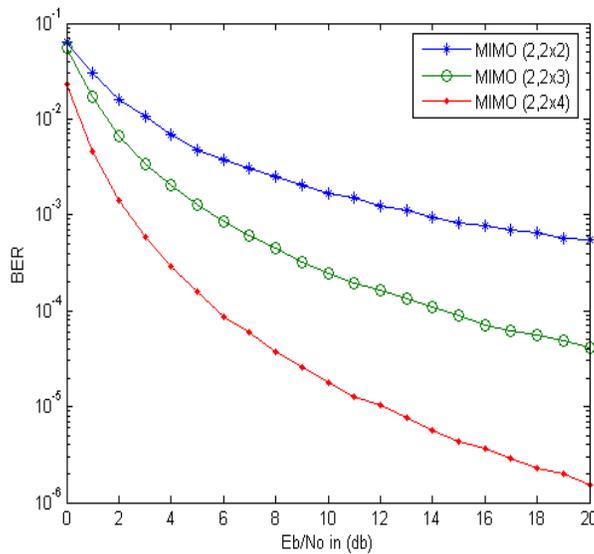


Figure3. Performance of MMSE precoding for MIMO system with  $\sigma_e^2 = 0.1$

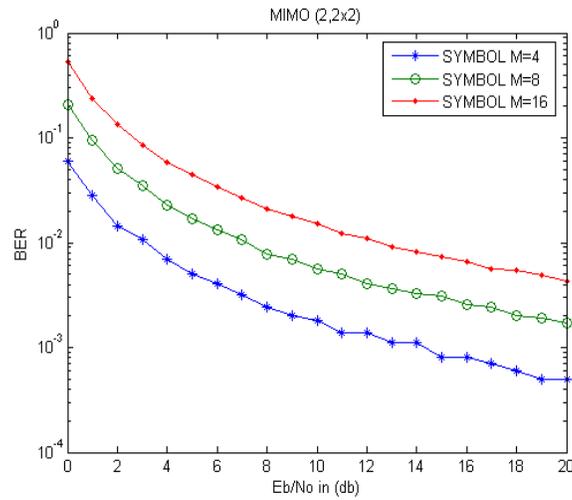


Figure4. Performance of MMSE precoding for MIMO system with  $\sigma_e^2 = 0.1$  for various symbol lengths

In figure 3, we plot the average BER for various configuration like (2,2×2),(2,2 ×3) , (2,2 ×4), i.e., two transmit antennas, two users ,each with two receive antennas, three receive antennas and four receive antennas respectively. It is noticed from the figure 3 that the slope of the (2, 2 ×3) configuration is larger than the configuration of (2, 2 ×2). Similarly, the performance of the (2, 2 ×4) outperforms (2, 2 ×3) and (2, 2 ×2). Hence it is evidenced that the simple receive diversity is achieved for MIMO channel with channel mismatch error. The BER performance of our proposed system is analyzed for various symbol lengths in figure 4. The BER performance is degraded if the length of the symbol is increased. Hence, it is evidenced from the above simulation that our proposed system and the precoder equation derived in (8) are validated.

## 5. CONCLUSION

In this paper we proposed a MMSE based pre-processing at the base station of a multiuser MIMO downlink channel. The proposed pre-processing techniques consider channel estimation errors as an integral part of the system design. Here, we assume that the channel is imperfect and we derived an analytic solution for optimized transmit scheme by minimizing the mean square for MIMO systems with a simple receive structure for the imperfect channel state information. Compared to previously propose pre-processing techniques such as for MISO pre-processing coding with channel estimation error as an integral part of the system design, it was shown that the proposed technique achieves an improvement in the average BER of the system for all values of  $E_b/n_0$ . Also, this proposed pre-processing technique provides simple receive diversity technique. Thus the receiver complexity can be reduced. This work can be extended for correlated channel.

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