ANTI-SYNCHRONIZATION OF HYPERCHAOTIC XU SYSTEMS VIA SLIDING MODE CONTROL

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ABSTRACT

This paper investigates the anti-synchronization of identical hyperchaotic Xu systems (Xu, Cai and Zheng, 2009) via sliding mode control. The stability results derived in this paper for the anti-synchronization of identical hyperchaotic Xu systems are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve anti- synchronization of the identical hyperchaotic Xu systems. Numerical simulations are shown to illustrate and validate the anti-synchronization schemes derived in this paper for the identical hyperchaotic Xu systems.

KEYWORDS

Sliding Mode Control, Anti-Synchronization, Hyperchaotic Systems, Hyperchaotic Xu System.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is usually known as the *butterfly effect* [1].

Synchronization of chaotic systems is a phenomenon which may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

Hyperchaotic system is usually defined as a chaotic system with more than one positive Lyapunov exponent. The first hyperchaotic system was discovered by O.E. Rössler (1979).
Since hyperchaotic system has the characteristics of high capacity, high security and high efficiency, it has the potential of broad applications in nonlinear circuits, secure communications, lasers, neural networks, biological systems and so on.

Thus, the studies on hyperchaotic systems, viz. control, synchronization and circuit implementation are very challenging problems in the chaos literature.

In the chaos literature, the *master-slave* formalism is used for the chaos synchronization problem. If a certain chaotic system is called the *master* system and another chaotic system is called the *slave* system, then the anti-synchronization problem is to use the output of the master system to control the slave system so that the sum of the states of the master and slave systems converges to zero asymptotically.

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The seminal work on chaos synchronization problem is due to Pecora and Carroll ([2], 1990). Chaos theory has been applied to a variety of fields such as physical systems [3], chemical systems [4], ecological systems [5], secure communications [6-8], etc.

In the last two decades, various schemes have been successfully applied for chaos synchronization such as PC method [2], OGY method [9], active control method [10-14], adaptive control method [15-20], time-delay feedback method [21], backstepping design method [22], sampled-data feedback method [23], etc.

In this paper, we derive new results based on the sliding mode control [24-26] for the anti-synchronization of identical hyperchaotic Xu systems ([27], Xu, Cai and Zheng, 2009). In robust control systems, the sliding mode control method is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and external disturbances.

2. Problem Statement and Our Methodology using SMC

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system.

We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system and $u \in R^m$ is the controller to be designed.
If we define the anti-synchronization error as

$$e = y + x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = Ae + \eta(x, y) + u, \tag{4}$$

where

$$\eta(x, y) = f(y) + f(x) \tag{5}$$

The objective of the anti-synchronization problem is to find a controller u such that

$$\lim_{t\to\infty} \|e(t)\| = 0 \quad \text{for all } e(0) \in R^n.$$

To solve this problem, we first define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.
Substituting (5) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, we replace the original anti-synchronization problem with an equivalent problem of stabilizing the zero solution of the system (7) by a suitable choice of the sliding mode control.
In the sliding mode control, we define the variable

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n \tag{8}$$

where $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \left\{ x \in R^n \mid s(e) = 0 \right\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\left[Ae + Bv \right] = 0 \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{\text{eq}}(t) = -(CB)^{-1} CA\, e(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

When we substitute (12) into the error dynamics (7), we get the closed-loop error dynamics as

$$\dot{e} = \left[I - B(CB)^{-1} C \right] Ae \tag{13}$$

The row vector C is chosen in such a way that the system matrix of the controlled dynamics (13) given by $\left[I - B(CB)^{-1} C \right] A$ is Hurwitz, *i.e.* it has all eigenvalues with negative real parts. Then the linear system (13) is globally asymptotically stable.

To design the sliding mode controller for (7), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\,\mathrm{sgn}(s) - k\,s \tag{14}$$

where $\mathrm{sgn}(\cdot)$ denotes the sign function and the gains $q > 0$, $k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (11) and (14), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\mathrm{sgn}(s)\right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q\right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q\right], & \text{if } s(e) < 0 \end{cases} \tag{16}$$

Theorem 2.1. The master system (1) and the slave system (2) are globally and asymptotically anti-synchronized for all initial conditions $x(0), y(0) \in R^n$ by the feedback control law

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ is defined by (15) and B is a column vector such that (A, B) is controllable. Also, the sliding mode gains k, q are positive constants.

Proof. First, we note that substituting (17) and (15) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\mathrm{sgn}(s)\right] \tag{18}$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2}s^2(e) \tag{19}$$

which is a positive definite function on R^n.

Differentiating V along the trajectories of (18) or the equivalent dynamics (14), we get

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\mathrm{sgn}(s)s \tag{20}$$

which is a negative definite function on R^n.

This calculation shows that V is a globally defined, positive definite, Lyapunov function for the error dynamics (18), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory [28], it is immediate that the error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

This means that for all initial conditions $e(0) \in R^n$, we have

$$\lim_{t\to\infty} \|e(t)\| = 0$$

Hence, it follows that the master system (1) and the slave system (2) are globally and asymptotically anti-synchronized for all initial conditions $x(0), y(0) \in R^n$.
This completes the proof.

3. ANTI-SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC XU SYSTEMS VIA SLIDING MODE CONTROL

3.1 Theoretical Results

In this section, new results on anti-synchronization of identical hyperchaotic Xu systems ([27], Xu et al. 2009) have been derived by means of applying Theorem 2.1 derived in Section 2.
Thus, the master system is described by the hyperchaotic Xu dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= bx_1 + fx_1x_3 \\
\dot{x}_3 &= -cx_3 - \varepsilon x_1x_2 \\
\dot{x}_4 &= -dx_4 + x_1x_3
\end{aligned} \tag{21}$$

where x_1, x_2, x_3, x_4 are state variables and $a, b, c, d, \varepsilon, f$ are positive, constant parameters of the system.

The Xu system (21) is *hyperchaotic* when the parameters are chosen as

$$a = 10,\ \ b = 40,\ c = 2.5,\ \ d = 2,\ \ \varepsilon = 1 \ \text{ and } \ f = 16$$

Figure 1 illustrates the phase portrait of the hyperchaotic Xu system.

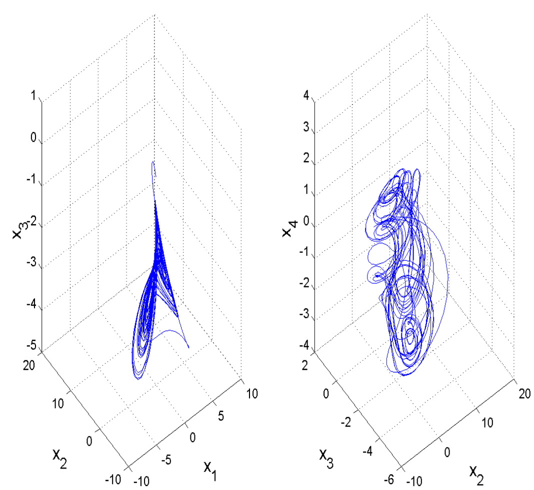

Figure 1. Phase Portrait of the Hyperchaotic Xu System

The slave system is described by the controlled hyperchaotic Xu dynamics

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= by_1 + fy_1 y_3 + u_2 \\
\dot{y}_3 &= -cy_3 - \varepsilon y_1 y_2 + u_3 \\
\dot{y}_4 &= -dy_4 + y_1 y_3 + u_4
\end{aligned} \tag{22}$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are the controllers to be designed.
The chaos *anti-synchronization error* is defined by

$$\begin{aligned}
e_1 &= y_1 + x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 + x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{23}$$

The error dynamics is easily obtained as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= be_1 + f(y_1 y_3 + x_1 x_3) + u_2 \\
\dot{e}_3 &= -ce_3 - \varepsilon(y_1 y_2 + x_1 x_2) + u_3 \\
\dot{e}_4 &= -de_4 + y_1 y_3 + x_1 x_3 + u_4
\end{aligned} \tag{24}$$

We write the error dynamics (24) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

Where

$$A = \begin{bmatrix} -a & a & 0 & 1 \\ b & 0 & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -d \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} 0 \\ f(y_1 y_3 + x_1 x_3) \\ -\varepsilon(y_1 y_2 + x_1 x_2) \\ y_1 y_3 + x_1 x_3 \end{bmatrix} \text{ and } \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}. \tag{26}$$

First, we set u as

$$u = -\eta(x, y) + Bv \tag{27}$$

where B is chosen such that (A, B) is controllable.
We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{28}$$

In the hyperchaotic case, the parameter values are taken as

$$a = 10,\quad b = 40,\quad c = 2.5,\quad d = 2,\quad \varepsilon = 1 \quad \text{and} \quad f = 16$$

The sliding mode variable is selected as

$$s = Ce = \begin{bmatrix} 9 & 1 & 1 & -9 \end{bmatrix} e = 9e_1 + e_2 + e_3 - 9e_4 \tag{29}$$

which makes the sliding mode state equation asymptotically stable.
We choose the sliding mode gains as

$$k = 6 \text{ and } q = 0.2.$$

Using Eq. (15), we can obtain the control $v(t)$ as

$$v(t) = -2e_1 - 48e_2 - 1.75e_3 + 13.5e_4 - 0.1\,\mathrm{sgn}(s) \tag{30}$$

Thus, by Theorem 2.1, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{31}$$

where $\eta(x, y), B$ and $v(t)$ are defined as in the equations (26), (28) and (30).

Theorem 3.1. The identical hyperchaotic Xu systems (21) and (22) are globally and asymptotically anti-synchronized for all initial conditions with the sliding mode controller u defined by (31).

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step

$$h = 10^{-8}$$

is used to solve the hyperchaotic Xu systems (21) and (22) with the sliding mode controller u given by (31) using MATLAB.

In the hyperchaotic case, the parameter values are given by

$$a = 10,\quad b = 40,\quad c = 2.5,\quad d = 2,\quad \varepsilon = 1 \quad \text{and} \quad f = 16$$

The sliding mode gains are chosen as

$$k = 6 \quad \text{and} \quad q = 0.2.$$

The initial values of the master system (21) are taken as

$$x_1(0) = 4,\; x_2(0) = 6,\quad x_3(0) = -7,\quad x_4(0) = -14$$

The initial values of the slave system (22) are taken as

$$y_1(0) = 8,\; y_2(0) = 15,\; y_3(0) = -6,\; y_4(0) = 12$$

Figure 2 illustrates the anti-synchronization of the identical hyperchaotic Xu systems (21) and (22).

Figure 3 illustrates the time-history of the synchronization errors e_1, e_2, e_3, e_4.

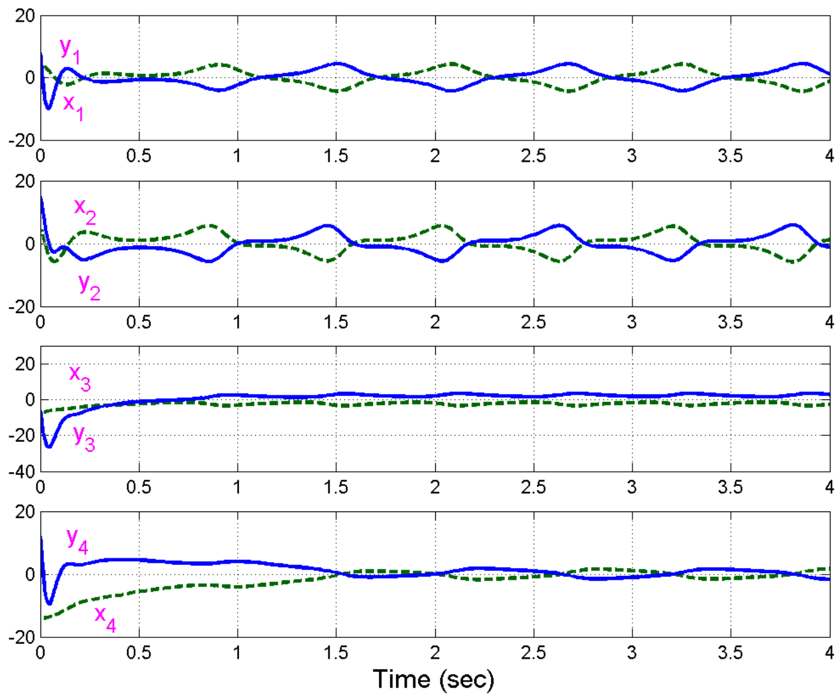

Figure 2. Anti-Synchronization of Identical Hyperchaotic Xu Systems

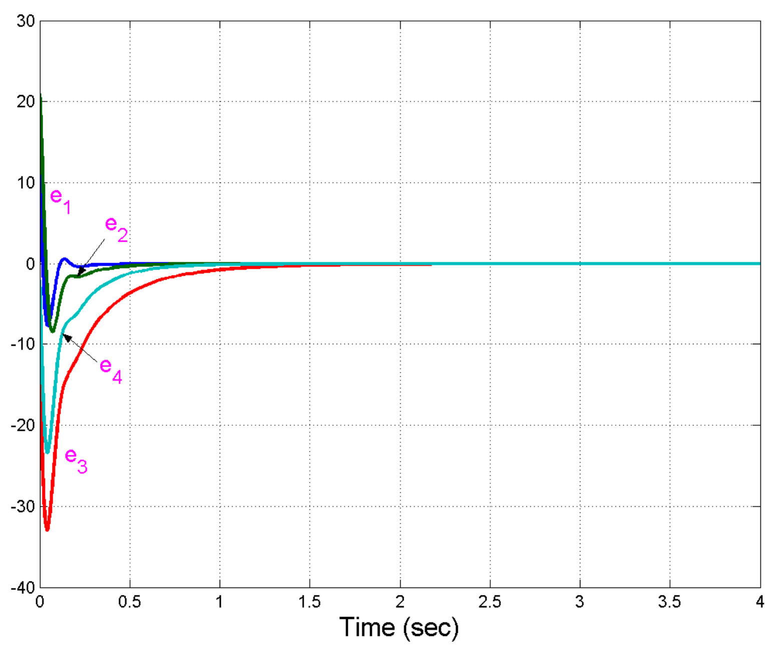

Figure 3. Time-History of the Anti-Synchronization Error

4. CONCLUSIONS

In this paper, we have deployed sliding mode control (SMC) to achieve anti-synchronization for the identical hyperchaotic Xu systems (2009). Our anti-synchronization results for the identical hyperchaotic Xu systems have been proved using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve anti-synchronization for the identical hyperchaotic Xu systems. Numerical simulations are also shown to illustrate the effectiveness of the anti-synchronization results derived in this paper using the sliding mode control.

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