ADAPTIVESYNCHRONIZER DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC ZHENGAND HYPERCHAOTIC YU SYSTEMS

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ABSTRACT

This paper derives new adaptive synchronizers for the hybrid synchronization of hyperchaotic Zheng systems (2010) and hyperchaotic Yu systems (2012). In the hybrid synchronization design of master and slave systems, one part of the systems, viz. their odd states, are completely synchronized (CS), while the other part, viz. their even states, are completely anti-synchronized (AS) so that CS and AS co-exist in the process of synchronization. The research problem gets even more complicated, when the parameters of the hyperchaotic systems are not known and we handle this complicate problem using adaptive control. The main results of this research work are established via adaptive control theory andLyapunov stability theory. MATLAB plotsusing classical fourth-order Runge-Kutta method have been depictedfor the new adaptive hybrid synchronization results for the hyperchaotic Zheng and hyperchaotic Yu systems.

KEYWORDS

Hybrid Synchronization, Adaptive Control, Chaos, Hyperchaos, Hyperchaotic Systems.

1. INTRODUCTION

Since thediscovery by the German scientist,O.E.Rössler ([1], 1979), hyperchaotic systems have found many applicationsin areas like neural networks [2],oscillators [3], communication [4-5], encryption [6], synchronization [7], etc. In chaos theory, hyperchaotic system is usually defined as a chaotic system having two or more positive Lyapunov exponents. Hyperchaotic systems have many attractive features like high efficiency, high capacity, high security, etc.

For the synchronization of chaotic systems, there are many methods available in the chaos literature like OGY method [8], PC method [9],backstepping method [10-12], sliding control method [13-15], active control method [16-17], adaptive control method [18-19], sampled-data feedback control [20], time-delay feedback method [21], etc.

In the hybrid synchronization of a pair of chaotic systems called the *master* and *slave* systems, one part of the systems, viz. the odd states, are completely synchronized (CS), while the other part of the systems, viz. the even states, are anti-synchronized so that CS and AS co-exist in the process of synchronization of the two systems.

This paper focuses upon adaptive controller design for the hybrid synchronization of hyperchaotic Zheng systems ([22], 2010) and hyperchaotic Yusystems ([23], 2012) with unknown parameters.

The main results derived in this paper have been proved using adaptive control theory [24] andLyapunov stability theory [25]..

2. ADAPTIVE CONTROL METHODOLOGYFOR HYBRID SYNCHRONIZATION

The *master system* is described by the chaotic dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part. The *slave system* is described by the chaotic dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where B is the $n \times n$ matrix of the system parameters and $g : R^n \to R^n$ is the nonlinear part For the pair of chaotic systems (1) and (2), the *hybrid synchronization error*is defined as

$$e_i = \begin{cases} y_i - x_i, & \text{if } i \text{ is odd} \\ y_i + x_i, & \text{if } i \text{ is even} \end{cases} \tag{3}$$

The error dynamics is obtained as

$$\dot{e}_i = \begin{cases} \sum_{j=1}^{n} (b_{ij} y_j - a_{ij} x_j) + g_i(y) - f_i(x) + u_i & \text{if } i \text{ is odd} \\ \sum_{j=1}^{n} (b_{ij} y_j + a_{ij} x_j) + g_i(y) + f_i(x) + u_i & \text{if } i \text{ is even} \end{cases} \tag{4}$$

The design goal is to find a feedback controller u so that

$$\lim_{t \to \infty} \| e(t) \| = 0 \text{ for all } e(0) \in R^n \tag{5}$$

Using the matrix method, we consider a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite matrix. It is noted that $V : R^n \to R$ is a positive definite function.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [25], the error dynamics (4) is globally exponentially stable. Hence, the states of the chaotic systems (1) and (2) will be globally and exponentially hybrid synchronized for all initial conditions $x(0), y(0) \in R^n$.When the system parameters are unknown, we use estimates for them and find a parameter update law using Lyapunov approach.

3. 4-D HYPERCHAOTIC SYSTEMS

The 4-D hyperchaotic Zhengsystem ([22], 2010) has the dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= bx_1 + cx_2 + x_1x_3 + x_4 \\ \dot{x}_3 &= -x_1^2 - rx_3 \\ \dot{x}_4 &= -dx_2 \end{aligned} \tag{8}$$

where a, b, c, r, d are constant, positive parameters of the system.

The 4-D Zheng system (8) exhibits a hyperchaotic attractor for the parametric values

$$a = 20, \quad b = 14, \quad c = 10.6, \quad d = 4, \quad r = 2.8 \tag{9}$$

The Lyapunov exponents of the system (8) for the parametric values in (9) are

$$L_1 = 1.8892, \quad L_2 = 0.2268, \quad L_3 = 0, \quad L_4 = -14.4130 \tag{10}$$

Since there are two positive Lyapunov exponents in (10), the Zheng system (8) is hyperchaotic for the parametric values (9).

The strange attractor of the hyperchaotic Zheng system is displayed in Figure 1.

The 4-D hyperchaotic Yu system ([23], 2012) has the dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \beta x_1 - x_1x_3 + \gamma x_2 + x_4 \\ \dot{x}_3 &= e^{x_1x_2} - \delta x_3 \\ \dot{x}_4 &= -\varepsilon x_1 \end{aligned} \tag{11}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are constant, positive parameters of the system.

The 4-D Yu system (11) exhibits a hyperchaotic attractor for the parametric values

$$\alpha = 10, \quad \beta = 40, \quad \gamma = 1, \quad \delta = 3, \quad \varepsilon = 8 \tag{12}$$

The Lyapunov exponents of the system (11) for the parametric values in (12) are

$$L_1 = 1.6877, \quad L_2 = 0.1214, \quad L_3 = 0, \quad L_4 = -13.7271 \tag{13}$$

Since there are two positive Lyapunov exponents in (13), theYusystem (11) is hyperchaotic for the parametric values (12).

The strange attractor of the hyperchaotic Yu system is displayed in Figure 2.

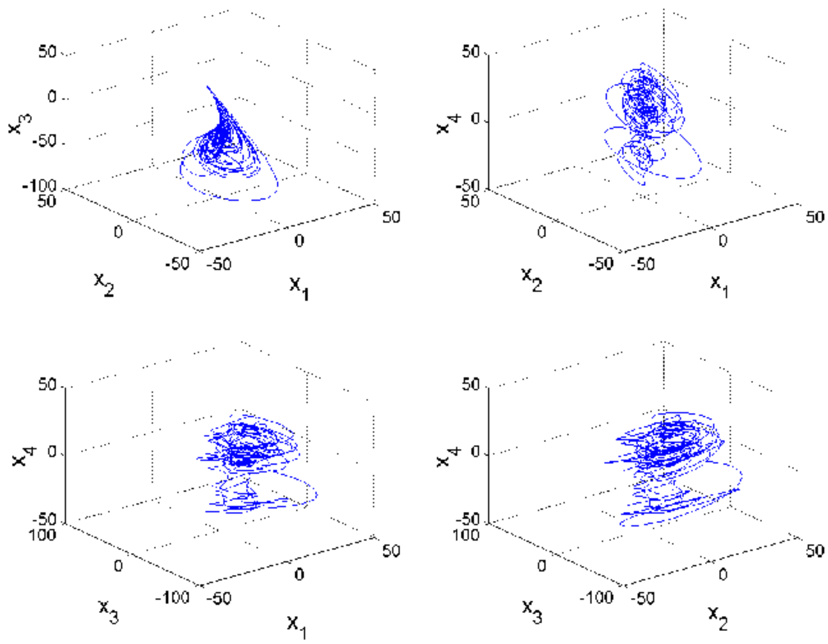

Figure 1. The State Portrait of the HyperchaoticZhengSystem

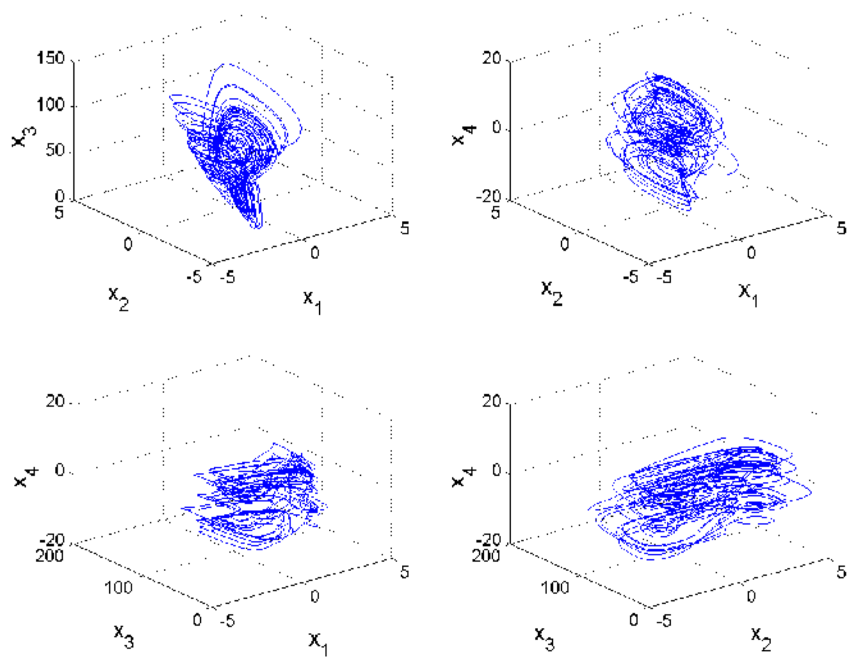

Figure 2. The State Portrait of the Hyperchaotic Yu System

4. ADAPTIVECONTROL DESIGN FOR THE HYBRIDSYNCHRONIZATION OF HYPERCHAOTIC ZHENG SYSTEMS

In this section, we design an adaptivesynchronizer for the hybrid synchronization of two identical hyperchaotic Zheng systems (2010) with unknown parameters.

The hyperchaotic Zheng system is taken as the master system, whose dynamics isgiven by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4\\ \dot{x}_2 &= bx_1 + cx_2 + x_1x_3 + x_4\\ \dot{x}_3 &= -x_1^2 - rx_3\\ \dot{x}_4 &= -dx_2\end{aligned} \tag{14}$$

where a, b, c, d, r are unknown parameters of the system and $x \in R^4$ is the state of the system.

The hyperchaotic Zheng system is also taken as the slave system, whose dynamics is given by

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1\\ \dot{y}_2 &= by_1 + cy_2 + y_1y_3 + y_4 + u_2\\ \dot{y}_3 &= -y_1^2 - ry_3 + u_3\\ \dot{y}_4 &= -dy_2 + u_4\end{aligned} \tag{15}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the adaptivecontrollers to be designed using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{r}(t)$ of the unknown parameters a, b, c, d, r, respectively.

For the hybrid synchronization, the error e is defined as

$$e_1 = y_1 - x_1,\ e_2 = y_2 + x_2,\ e_3 = y_3 - x_3,\ e_4 = y_4 + x_4 \tag{16}$$

A simple calculation gives the error dynamics

$$\begin{aligned}\dot{e}_1 &= a(y_2 - x_2 - e_1) + y_4 - x_4 + u_1\\ \dot{e}_2 &= b(y_1 + x_1) + ce_2 + e_4 + y_1y_3 + x_1x_3 + u_2\\ \dot{e}_3 &= -re_3 - y_1^2 + x_1^2 + u_3\\ \dot{e}_4 &= -de_2 + u_4\end{aligned} \tag{17}$$

Next, we choose a nonlinear controller for achieving hybrid synchronization as

$$\begin{aligned}u_1 &= -\hat{a}(t)(y_2 - x_2 - e_1) - y_4 + x_4 - k_1e_1\\ u_2 &= -\hat{b}(t)(y_1 + x_1) - \hat{c}(t)e_2 - e_4 - y_1y_3 - x_1x_3 - k_2e_2\\ u_3 &= \hat{r}(t)e_3 + y_1^2 - x_1^2 - k_3e_3\\ u_4 &= \hat{d}(t)e_2 - k_4e_4\end{aligned} \tag{18}$$

In Eq. (18), $k_i,\ (i=1,2,3,4)$ are positive gains and $\hat{a}(t),\hat{b}(t),\hat{c}(t),\hat{d}(t),\hat{r}(t)$ are estimates of the unknown parameters a,b,c,d,r, respectively.

By the substitution of (18) into (17), the error dynamics is determined as

$$\begin{aligned}
\dot{e}_1 &= (a-\hat{a}(t))(y_2-x_2-e_1)-k_1e_1 \\
\dot{e}_2 &= (b-\hat{b}(t))(y_1+x_1)+(c-\hat{c}(t))e_2-k_2e_2 \\
\dot{e}_3 &= -(r-\hat{r}(t))e_3-k_3e_3 \\
\dot{e}_4 &= -(d-\hat{d}(t))e_2-k_4e_4
\end{aligned} \tag{19}$$

Next, we define the parameter estimation errors as

$$e_a(t)=a-\hat{a}(t),\ e_b(t)=b-\hat{b}(t),\ e_c(t)=c-\hat{c}(t),\ e_d(t)=d-\hat{d}(t),\ e_r(t)=r-\hat{r}(t) \tag{20}$$

Differentiating (20) with respect to t, we get

$$\dot{e}_a(t)=-\dot{\hat{a}}(t),\ \dot{e}_b(t)=-\dot{\hat{b}}(t),\ \dot{e}_c(t)=-\dot{\hat{c}}(t),\ \dot{e}_d(t)=-\dot{\hat{d}}(t),\ \dot{e}_r(t)=-\dot{\hat{r}}(t) \tag{21}$$

In view of (20), we can simplify the error dynamics (19) as

$$\begin{aligned}
\dot{e}_1 &= e_a(y_2-x_2-e_1)-k_1e_1 \\
\dot{e}_2 &= e_b(y_1+x_1)+e_ce_2-k_2e_2 \\
\dot{e}_3 &= -e_re_3-k_3e_3 \\
\dot{e}_4 &= -e_de_2-k_4e_4
\end{aligned} \tag{22}$$

We take the quadratic Lyapunov function

$$V=\frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_4^2+e_a^2+e_b^2+e_c^2+e_d^2+e_r^2\right), \tag{23}$$

Which is a positive definite function on R^9.

When we differentiate (22) along the trajectories of (19) and (21), we get

$$\begin{aligned}
\dot{V} = &-k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2+e_a\left[e_1(y_2-x_2-e_1)-\dot{\hat{a}}\right]+e_b\left[e_2(y_1+x_1)-\dot{\hat{b}}\right] \\
&+e_c\left[e_2^2-\dot{\hat{c}}\right]+e_d\left[-e_2e_4-\dot{\hat{d}}\right]+e_r\left[-e_3^2-\dot{\hat{r}}\right]
\end{aligned} \tag{24}$$

In view of Eq. (24), we take the parameter update law as

$$\begin{aligned}
&\dot{\hat{a}}=e_1(y_2-x_2-e_1)+k_5e_a, \quad \dot{\hat{b}}=e_2(y_1+x_1)+k_6e_b, \qquad \dot{\hat{c}}=e_2^2+k_7e_c \\
&\dot{\hat{d}}=-e_2e_4+k_8e_d, \qquad\qquad \dot{\hat{r}}=-e_3^2+k_9e_r
\end{aligned} \tag{25}$$

Theorem 4.1 The adaptive control law (18) along with the parameter update law (25), where $k_i,(i=1,2,\ldots,9)$ are positive gains, achieves global and exponential hybrid synchronization of the identical hyperchaotic Zheng systems (14) and (15), where $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t), \hat{r}(t)$ are estimates of the unknown parameters $a, b, c, d, r,$ respectively. In addition, the parameter estimation errors e_a, e_b, e_c, e_d, e_r converge to zero exponentially for all initial conditions.

Proof. We prove the above result using Lyapunov stability theory [25].

Substituting the parameter update law (25) into (24), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_r^2 \tag{26}$$

which is a negative definite function on R^9.

This shows that the hybrid synchronization errors $e_1(t), e_2(t), e_3(t), e_4(t)$ and the parameter estimation errors $e_a(t), e_b(t), e_c(t), e_d(t), e_r(t)$ are globally exponentially stable for all initial conditions.This completes the proof. ■

Next, we use MATLAB to demonstrate our hybrid synchronization results.

The classical fourthorder Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Zheng systems (14) and (15) with the adaptive nonlinear controller(18) and the parameter update law (25). The feedback gains arechosen as $k_i = 5, \ (i = 1, 2, \ldots, 9).$

The parameters of the hyperchaotic Zheng systems are taken as in the hyperchaotic case, *i.e.*

$$a = 20, \quad b = 14, \quad c = 10.6, \quad d = 4, \quad r = 2.8$$

For simulations, the initial conditions of the hyperchaotic Zheng system (14) are chosen as

$$x_1(0) = 24, \quad x_2(0) = -15, \quad x_3(0) = -6, \quad x_4(0) = 18$$

Also, the initial conditions of the hyperchaotic Zheng system (15) are chosen as

$$y_1(0) = 12, \quad y_2(0) = -9, \quad y_3(0) = 26, \quad y_4(0) = -6$$

Also, the initial conditions of the parameter estimates are chosen as

$$\hat{a}(0) = 9, \quad \hat{b}(0) = -7, \quad \hat{c}(0) = 8, \quad \hat{d}(0) = 2, \quad \hat{r}(0) = -5$$

Figure 3 depicts the hybrid synchronization of the identical hyperchaoticZheng systems.

Figure 4 depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4.

Figure 5 depicts the time-history of the parameter estimation errors e_a, e_b, e_c, e_d, e_r.

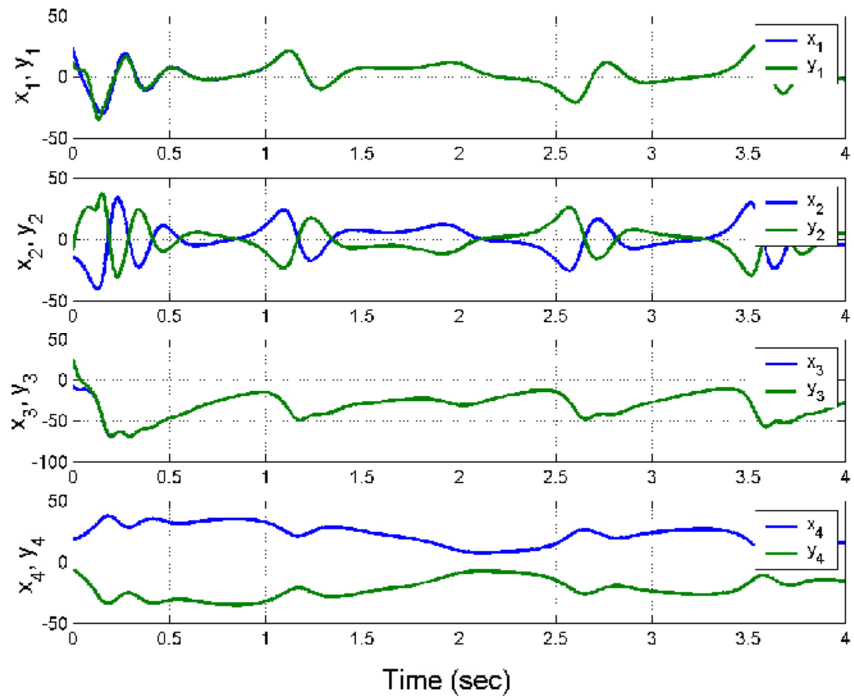

Figure 3.Hybrid Synchronization of Identical Hyperchaotic Zheng Systems

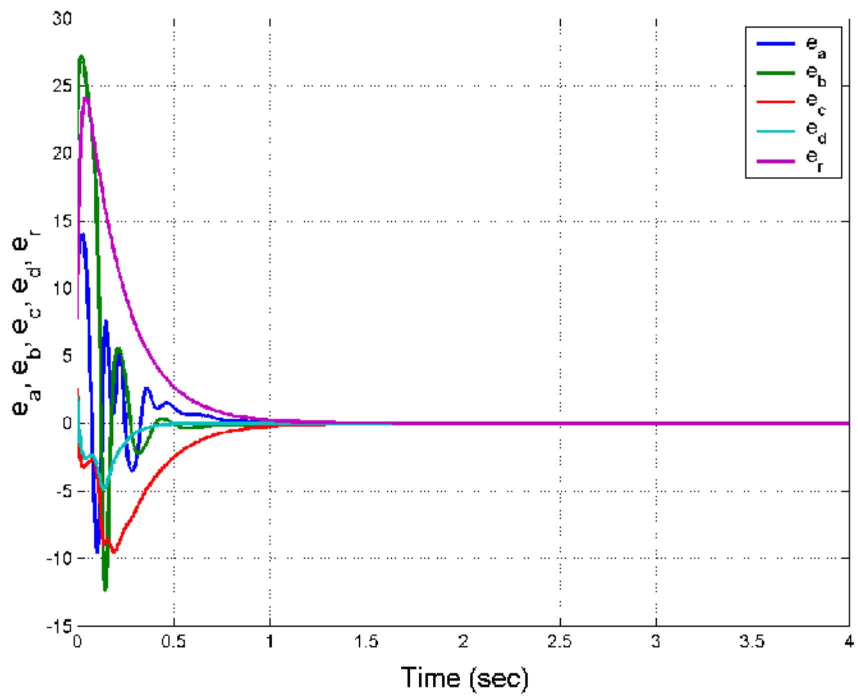

Figure 4. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

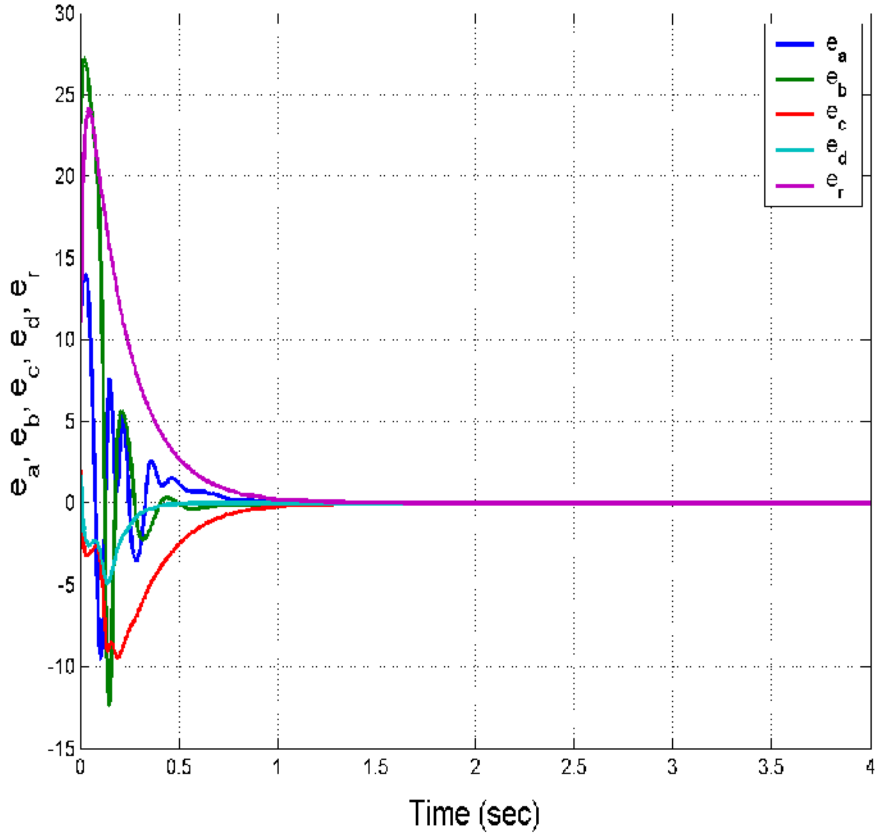

Figure 5. Time-History of the Parameter Estimation Errors e_a, e_b, e_c, e_d, e_r

5. ADAPTIVE CONTROLLER DESIGN FOR THE HYBRID SYNCHRONIZATION DESIGN OF HYPERCHAOTIC YU SYSTEMS

In this section, we design an adaptive controller for the hybrid synchronization of two identical hyperchaotic Yusystems (2012) with unknown parameters.

The hyperchaotic Yusystem is taken as the master system, whose dynamics is given by

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - x_1) \\
\dot{x}_2 &= \beta x_1 - x_1 x_3 + \gamma x_2 + x_4 \\
\dot{x}_3 &= e^{x_1 x_2} - \delta x_3 \\
\dot{x}_4 &= -\varepsilon x_1
\end{aligned} \tag{27}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are unknown parameters of the system and $x \in R^4$ is the state of the system.

The hyperchaotic Yu system is also taken as the slave system, whose dynamics is given by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \beta y_1 - y_1 y_3 + \gamma y_2 + y_4 + u_2 \\ \dot{y}_3 &= e^{y_1 y_2} - \delta y_3 + u_3 \\ \dot{y}_4 &= -\varepsilon y_1 + u_4\end{aligned} \tag{28}$$

Where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the adaptivecontrollers to be designed using estimates $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t), \hat{\varepsilon}(t)$ of the unknown parameters $\alpha, \beta, \gamma, \delta, \varepsilon,$ respectively.

For the hybrid synchronization, the error e is defined as

$$\begin{aligned}e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4\end{aligned} \tag{29}$$

A simple calculation gives the error dynamics

$$\begin{aligned}\dot{e}_1 &= \alpha(y_2 - x_2 - e_1) + u_1 \\ \dot{e}_2 &= \beta(y_1 + x_1) + \gamma e_2 + e_4 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -\delta e_3 + e^{y_1 y_2} - e^{x_1 x_2} + u_3 \\ \dot{e}_4 &= -\varepsilon(y_1 + x_1) + u_4\end{aligned} \tag{30}$$

Next, we choose a nonlinear controller for achieving hybrid synchronization as

$$\begin{aligned}u_1 &= -\hat{\alpha}(t)(y_2 - x_2 - e_1) - k_1 e_1 \\ u_2 &= -\hat{\beta}(t)(y_1 + x_1) - \hat{\gamma}(t) e_2 - e_4 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 &= \hat{\delta}(t) e_3 - e^{y_1 y_2} + e^{x_1 x_2} - k_3 e_3 \\ u_4 &= \hat{\varepsilon}(t)(y_1 + x_1) - k_4 e_4\end{aligned} \tag{31}$$

In Eq. (31), k_i, $(i = 1, 2, 3, 4)$ are positive gains.

By the substitution of (31) into (30), the error dynamics is simplified as

$$\begin{aligned}\dot{e}_1 &= (\alpha - \hat{\alpha}(t))(y_2 - y_1) - k_1 e_1 \\ \dot{e}_2 &= (\beta - \hat{\beta}(t))(y_1 + x_1) + (\gamma - \hat{\gamma}(t)) e_2 - k_2 e_2 \\ \dot{e}_3 &= -(\delta - \hat{\delta}(t)) e_3 - k_3 e_3 \\ \dot{e}_4 &= -(\varepsilon - \hat{\varepsilon}(t))(y_1 + x_1) - k_4 e_4\end{aligned} \tag{32}$$

Next, we define the parameter estimation errors as

$$\begin{aligned} e_\alpha(t) &= \alpha - \hat{\alpha}(t) \\ e_\beta(t) &= \beta - \hat{\beta}(t) \\ e_\gamma(t) &= \gamma - \hat{\gamma}(t) \\ e_\delta(t) &= \delta - \hat{\delta}(t) \\ e_\varepsilon(t) &= \varepsilon - \hat{\varepsilon}(t) \end{aligned} \tag{33}$$

Differentiating (33) with respect to t, we get

$$\dot{e}_\alpha(t) = -\dot{\hat{\alpha}}(t),\ \dot{e}_\beta(t) = -\dot{\hat{\beta}}(t),\ \dot{e}_\gamma(t) = -\dot{\hat{\gamma}}(t),\ \dot{e}_\delta(t) = -\dot{\hat{\delta}}(t),\ \ \dot{e}_\varepsilon(t) = -\dot{\hat{\varepsilon}}(t) \tag{34}$$

In view of (33), we can simplify the error dynamics (32) as

$$\begin{aligned} \dot{e}_1 &= e_\alpha(y_2 - x_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_\beta(y_1 + x_1) + e_\gamma e_2 - k_2 e_2 \\ \dot{e}_3 &= -e_\delta e_3 - k_3 e_3 \\ \dot{e}_4 &= -e_\varepsilon(y_1 + x_1) - k_4 e_4 \end{aligned} \tag{35}$$

We take the quadratic Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 + e_\delta^2 + e_\varepsilon^2\right), \tag{36}$$

which is a positive definite function on R^9.

When we differentiate (35) along the trajectories of (32) and (33), we get

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_\alpha\left[e_1(y_2 - x_2 - e_1) - \dot{\hat{\alpha}}\right] + e_\beta\left[e_2(y_1 + x_1) - \dot{\hat{\beta}}\right] \\ &+ e_\gamma\left[e_2^2 - \dot{\hat{\gamma}}\right] + e_\delta\left[-e_3^2 - \dot{\hat{\delta}}\right] + e_\varepsilon\left[-e_4(y_1 + x_1) - \dot{\hat{\varepsilon}}\right] \end{aligned} \tag{37}$$

In view of Eq. (37), we take the parameter update law as

$$\begin{aligned} &\dot{\hat{\alpha}} = e_1(y_2 - x_2 - e_1) + k_5 e_\alpha, \quad \dot{\hat{\beta}} = e_2(y_1 + x_1) + k_6 e_\beta, \quad \dot{\hat{\gamma}} = e_2^2 + k_7 e_\gamma \\ &\dot{\hat{\delta}} = -e_3^2 + k_8 e_\delta, \qquad\qquad\quad \dot{\hat{\varepsilon}} = -e_4(y_1 + x_1) + k_9 e_\varepsilon \end{aligned} \tag{38}$$

Theorem 5.1 The adaptive control law (31) along with the parameter update law (38), where $k_i, (i = 1, 2, \ldots, 9)$ are positive gains, achieves global and exponential hybrid synchronization of the identical hyperchaotic Yu systems (27) and (28), where $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t), \hat{\varepsilon}(t)$ are estimates of the unknown parameters $\alpha, \beta, \gamma, \delta, \varepsilon,$ respectively. Moreover, all the parameter estimation errors converge to zero exponentially for all initial conditions.

Proof. We prove the above result using Lyapunov stability theory [25].

Substituting the parameter update law (38) into (37), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_\alpha^2 - k_6 e_\beta^2 - k_7 e_\gamma^2 - k_8 e_\delta^2 - k_9 e_\varepsilon^2 \tag{39}$$

which is a negative definite function on R^9.

This shows that the hybrid synchronization errors $e_1(t), e_2(t), e_3(t), e_4(t)$ and the parameter estimation errors $e_\alpha(t), e_\beta(t), e_\gamma(t), e_\delta(t), e_\varepsilon(t)$ are globally exponentially stable for all initial conditions.This completes the proof. ■

Next, we demonstrate our hybrid synchronization results with MATLAB simulations.

The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Yu systems (27) and (28) with the adaptive nonlinear controller(31) and the parameter update law (38). The feedback gains are taken as

$$k_i = 5, \ (i = 1, 2, \ldots, 9).$$

The parameters of the hyperchaotic Yu systems are taken as in the hyperchaotic case, *i.e.*

$$\alpha = 10, \ \ \beta = 40, \ \ \gamma = 1, \ \ \delta = 3, \ \ \varepsilon = 8$$

For simulations, the initial conditions of the hyperchaoticYu system (27) are chosen as

$$x_1(0) = 4, \ \ x_2(0) = -2, \ \ x_3(0) = 8, \ \ x_4(0) = -10$$

Also, the initial conditions of the hyperchaotic Yusystem (28) are chosen as

$$y_1(0) = 16, \ \ y_2(0) = 8, \ \ y_3(0) = 12, \ \ y_4(0) = -6$$

Also, the initial conditions of the parameter estimates are chosen as

$$\hat{\alpha}(0) = 17, \ \ \hat{\beta}(0) = -7, \ \ \hat{\gamma}(0) = 12, \ \ \hat{\delta}(0) = -5, \ \ \hat{\varepsilon}(0) = 6$$

Figure 6depicts the hybrid synchronization of the identical hyperchaoticYu systems.

Figure 7depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4.

Figure 8depicts the time-history of the parameter estimation errors $e_\alpha, e_\beta, e_\gamma, e_\delta, e_\varepsilon$.

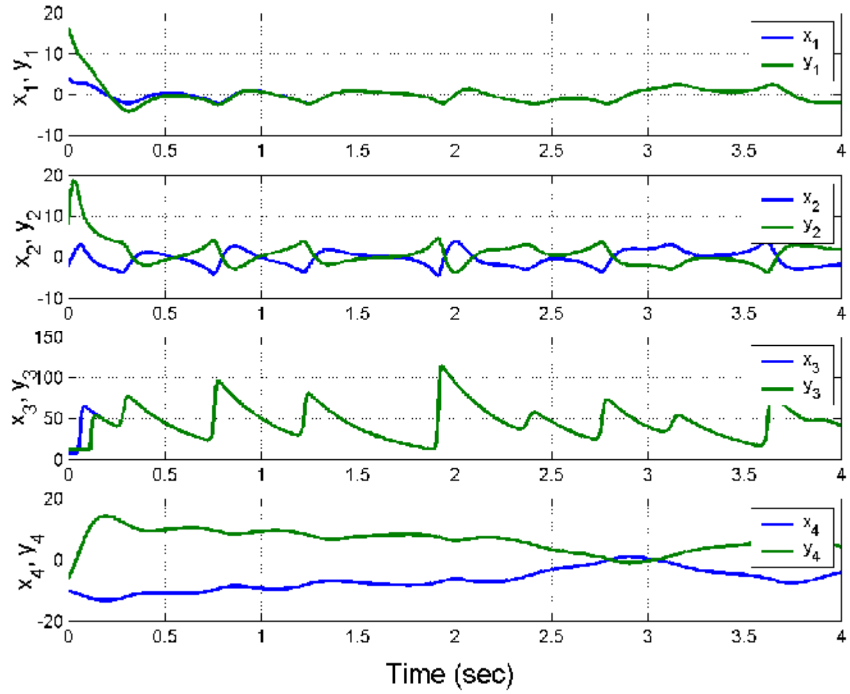

Figure 6.Hybrid Synchronization of Identical Hyperchaotic Yu Systems

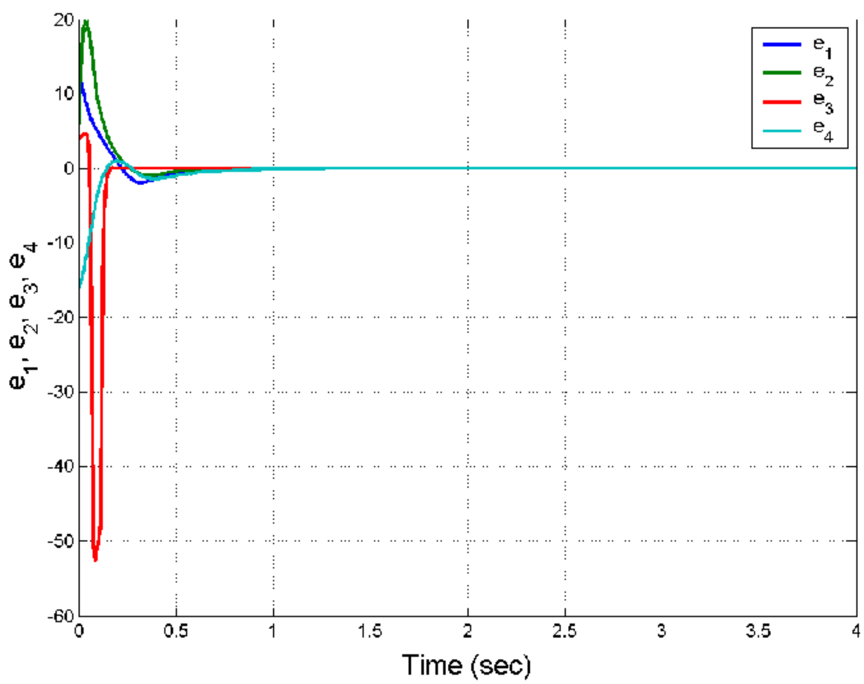

Figure 7. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

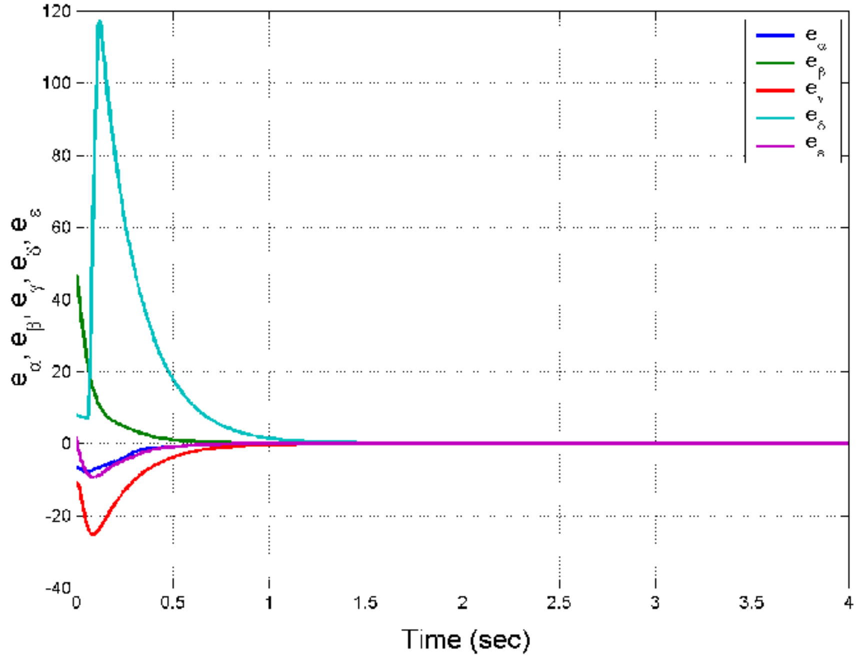

Figure 8. Time-History of the Parameter Estimation Errors $e_\alpha, e_\beta, e_\gamma, e_\delta, e_\varepsilon$

6. Adaptive Controller Design for the Hybrid Synchronization Design of Hyperchaotic Zheng and Hyperchaotic Yu Systems

In this section, we design an adaptive controller for the hybrid synchronization ofhyperchaoticZheng system (2010) and hyperchaotic Yusystem (2012) with unknown parameters.

The hyperchaotic Zhengsystem is taken as the master system, whose dynamics is given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= bx_1 + cx_2 + x_1x_3 + x_4 \\
\dot{x}_3 &= -x_1^2 - rx_3 \\
\dot{x}_4 &= -dx_2
\end{aligned} \tag{40}$$

where a, b, c, d, r are unknown parameters of the system.

The hyperchaotic Yu system is also taken as the slave system, whose dynamics is given by

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\
\dot{y}_2 &= \beta y_1 - y_1y_3 + \gamma y_2 + y_4 + u_2 \\
\dot{y}_3 &= e^{y_1y_2} - \delta y_3 + u_3 \\
\dot{y}_4 &= -\varepsilon y_1 + u_4
\end{aligned} \tag{41}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are unknown parametersand u_1, u_2, u_3, u_4 are the adaptivecontrollers.

For the hybrid synchronization, the error e is defined as

$$e_1 = y_1 - x_1,\ e_2 = y_2 + x_2,\ e_3 = y_3 - x_3,\ e_4 = y_4 + x_4 \tag{42}$$

A simple calculation gives the error dynamics

$$\begin{aligned}
\dot{e}_1 &= \alpha(y_2 - y_1) - a(x_2 - x_1) - x_4 + u_1 \\
\dot{e}_2 &= \beta y_1 + \gamma y_2 + e_4 + bx_1 + cx_2 - y_1 y_3 + x_1 x_3 + u_2 \\
\dot{e}_3 &= -\delta y_3 + rx_3 + e^{y_1 y_2} + x_1^2 + u_3 \\
\dot{e}_4 &= -\varepsilon y_1 - dx_2 + u_4
\end{aligned} \tag{43}$$

Next, we choose a nonlinear controller for achieving hybrid synchronization as

$$\begin{aligned}
u_1 &= -\hat{\alpha}(t)(y_2 - y_1) + \hat{a}(t)(x_2 - x_1) + x_4 - k_1 e_1 \\
u_2 &= -\hat{\beta}(t) y_1 - \hat{\gamma}(t) y_2 - e_4 - \hat{b}(t) x_1 - \hat{c}(t) x_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\
u_3 &= \hat{\delta}(t) y_3 - \hat{r}(t) x_3 - e^{y_1 y_2} - x_1^2 - k_3 e_3 \\
u_4 &= \hat{\varepsilon}(t) y_1 + \hat{d}(t) x_2 - k_4 e_4
\end{aligned} \tag{44}$$

where $k_i,\ (i = 1, 2, 3, 4)$ are positive gains.

By the substitution of (44) into (43), the error dynamics is obtained as

$$\begin{aligned}
\dot{e}_1 &= (\alpha - \hat{\alpha}(t))(y_2 - y_1) - (a - \hat{a}(t))(x_2 - x_1) - k_1 e_1 \\
\dot{e}_2 &= (\beta - \hat{\beta}(t)) y_1 + (\gamma - \hat{\gamma}(t)) y_2 + (b - \hat{b}(t)) x_1 + (c - \hat{c}(t)) x_2 - k_2 e_2 \\
\dot{e}_3 &= -(\delta - \hat{\delta}(t)) y_3 + (r - \hat{r}(t)) x_3 - k_3 e_3 \\
\dot{e}_4 &= -(\varepsilon - \hat{\varepsilon}(t)) y_1 - (d - \hat{d}(t)) x_2 - k_4 e_4
\end{aligned} \tag{45}$$

Next, we define the parameter estimation errors as

$$\begin{aligned}
& e_a(t) = a - \hat{a}(t),\ \ e_b(t) = b - \hat{b}(t),\ \ e_c(t) = c - \hat{c}(t),\ e_d(t) = d - \hat{d}(t) \\
& e_r(t) = r - \hat{r}(t),\ \ e_\alpha(t) = \alpha - \hat{\alpha}(t),\ e_\beta(t) = \beta - \hat{\beta}(t),\ e_\gamma(t) = \gamma - \hat{\gamma}(t) \\
& e_\delta(t) = \delta - \hat{\delta}(t),\ e_\varepsilon(t) = \varepsilon - \hat{\varepsilon}(t)
\end{aligned} \tag{46}$$

Differentiating (46) with respect to t, we get

$$\begin{aligned}
& \dot{e}_a(t) = -\dot{\hat{a}}(t),\ \dot{e}_b(t) = -\dot{\hat{b}}(t),\ \dot{e}_c(t) = -\dot{\hat{c}}(t),\ \dot{e}_d(t) = -\dot{\hat{d}}(t),\ \dot{e}_r(t) = -\dot{\hat{r}}(t) \\
& \dot{e}_\alpha(t) = -\dot{\hat{\alpha}}(t),\ \dot{e}_\beta(t) = -\dot{\hat{\beta}}(t),\ \dot{e}_\gamma(t) = -\dot{\hat{\gamma}}(t),\ \dot{e}_\delta(t) = -\dot{\hat{\delta}}(t),\ \ \dot{e}_\varepsilon(t) = -\dot{\hat{\varepsilon}}(t)
\end{aligned} \tag{47}$$

In view of (46), we can simplify the error dynamics (45) as

$$\begin{aligned}\dot{e}_1 &= e_\alpha(y_2 - y_1) - e_a(x_2 - x_1) - k_1 e_1 \\ \dot{e}_2 &= e_\beta y_1 + e_\gamma y_2 + e_b x_1 + e_c x_2 - k_2 e_2 \\ \dot{e}_3 &= -e_\delta y_3 + e_r x_3 - k_3 e_3 \\ \dot{e}_4 &= -e_\varepsilon y_1 - e_d x_2 - k_4 e_4\end{aligned} \tag{48}$$

We take the quadratic Lyapunov function

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_r^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 + e_\delta^2 + e_\varepsilon^2\right) \tag{49}$$

When we differentiate (48) along the trajectories of (45) and (46), we get

$$\begin{aligned}\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[-e_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[e_2 x_1 - \dot{\hat{b}}\right] + e_c\left[e_2 x_2 - \dot{\hat{c}}\right] \\ &+ e_d\left[-e_4 x_2 - \dot{\hat{d}}\right] + e_r\left[e_3 x_3 - \dot{\hat{r}}\right] + e_\alpha\left[e_1(y_2 - y_1) - \dot{\hat{\alpha}}\right] + e_\beta\left[e_2 y_1 - \dot{\hat{\beta}}\right] \\ &+ e_\gamma\left[e_2 y_2 - \dot{\hat{\gamma}}\right] + e_\delta\left[-e_3 y_3 - \dot{\hat{\delta}}\right] + e_\varepsilon\left[-e_4 y_1 - \dot{\hat{\varepsilon}}\right]\end{aligned} \tag{50}$$

In view of Eq. (50), we take the parameter update law as

$$\begin{aligned}&\dot{\hat{a}} = -e_1(x_2 - x_1) + k_5 e_a, \quad &&\dot{\hat{b}} = e_2 x_1 + k_6 e_b, \quad &&\dot{\hat{c}} = e_2 x_2 + k_7 e_c \\ &\dot{\hat{d}} = -e_4 x_2 + k_8 e_d, &&\dot{\hat{r}} = e_3 x_3 + k_9 e_r, &&\dot{\hat{\alpha}} = e_1(y_2 - y_1) + k_{10} e_\alpha \\ &\dot{\hat{\beta}} = e_2 y_1 + k_{11} e_\beta, &&\dot{\hat{\gamma}} = e_2 y_2 + k_{12} e_\gamma, &&\dot{\hat{\delta}} = -e_3 y_3 + k_{13} e_\delta \\ &\dot{\hat{\varepsilon}} = -e_4 y_1 + k_{14} e_\varepsilon\end{aligned} \tag{51}$$

Theorem 6.1 The adaptive control law (44) along with the parameter update law (51), where $k_i, (i = 1, 2, \ldots, 14)$ are positive gains, achieves global and exponential hybrid synchronization of the hyperchaotic Zheng system (40) hyperchaotic Yu system (41), where $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{d}(t)$, $\hat{r}(t)$, $\hat{\alpha}(t)$, $\hat{\beta}(t)$, $\hat{\gamma}(t)$, $\hat{\delta}(t)$, $\hat{\varepsilon}(t)$ are estimates of the unknown parameters a, b, c, d, r, $\alpha, \beta, \gamma, \delta, \varepsilon$, respectively. Moreover, all the parameter estimation errors converge to zero exponentially for all initial conditions.

Proof. We prove the above result using Lyapunov stability theory [25].Substituting the parameter update law (51) into (50), we get

$$\begin{aligned}\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 - k_9 e_r^2 \\ &- k_{10} e_\alpha^2 - k_{11} e_\beta^2 - k_{12} e_\gamma^2 - k_{13} e_\delta^2 - k_{14} e_\varepsilon^2\end{aligned} \tag{52}$$

which is a negative definite function on R^{14}.

This shows that the hybrid synchronization errors $e_1(t), e_2(t), e_3(t), e_4(t)$ and the parameter estimation errors $e_a(t), e_b(t), e_c(t), e_d(t), e_r(t)$, $e_\alpha(t), e_\beta(t), e_\gamma(t), e_\delta(t), e_\varepsilon(t)$ are globally exponentially stable for all initial conditions. This completes the proof. ■

For simulations, theclassical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Li systems (27) and (28) with the adaptive nonlinear controller(31) and the parameter update law (38). The feedback gains are taken as $k_i = 5,\ (i = 1, 2, \ldots, 14)$. The parameters of the hyperchaotic Zheng and hyperchaotic Yu systems are takenas $a = 20,\ b = 14,\ c = 10.6,\quad d = 4,\ r = 2.8,\ \alpha = 10,\ \beta = 40,\ \gamma = 1,\ \delta = 3$ and $\varepsilon = 8.$

For simulations, the initial conditions of the hyperchaotic Zheng system (40) are chosen as

$$x_1(0) = 4,\ \ x_2(0) = 9,\ \ x_3(0) = 1,\ \ x_4(0) = 4$$

Also, the initial conditions of the hyperchaotic Yu system (41) are chosen as

$$y_1(0) = 8,\ \ y_2(0) = 3,\ \ y_3(0) = 1,\ \ y_4(0) = 2$$

Also, the initial conditions of the parameter estimates are chosen as

$$\hat{a}(0) = 2, \hat{b}(0) = 6, \hat{c}(0) = 3, \hat{d}(0) = -3, \hat{r}(0) = -1, \hat{\alpha}(0) = 7, \hat{\beta}(0) = 4, \hat{\gamma}(0) = 9, \hat{\delta}(0) = 5, \hat{\varepsilon}(0) = 4$$

Figure 9depicts the hybrid synchronization of hyperchaoticZheng and hyperchaotic Yu systems. Figure 10depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4. Figure 11depicts the time-history of the parameter estimation errors e_a, e_b, e_c, e_d, e_r. Figure 12depicts the time-history of the parameter estimation errors $e_\alpha, e_\beta, e_\gamma, e_\delta, e_\varepsilon$.

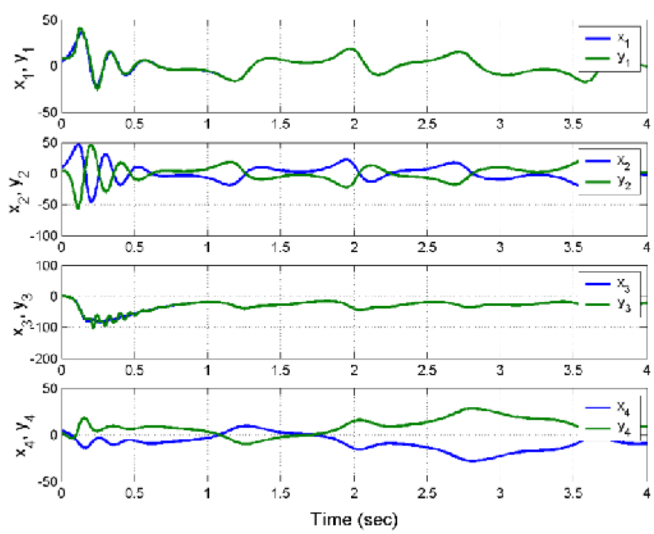

Figure 9.Hybrid Synchronization of Hyperchaotic Xu and Lu Systems

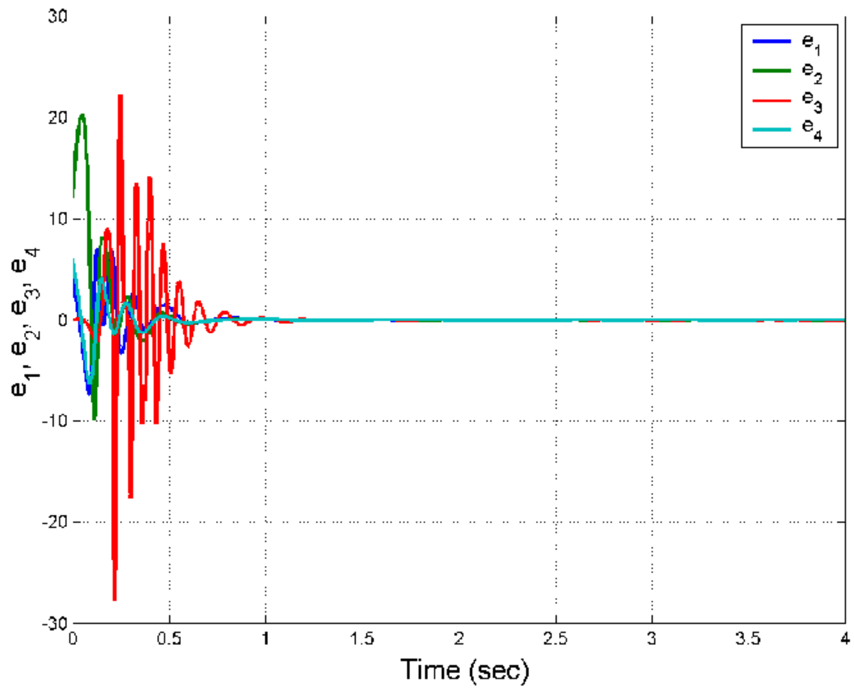

Figure 10. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

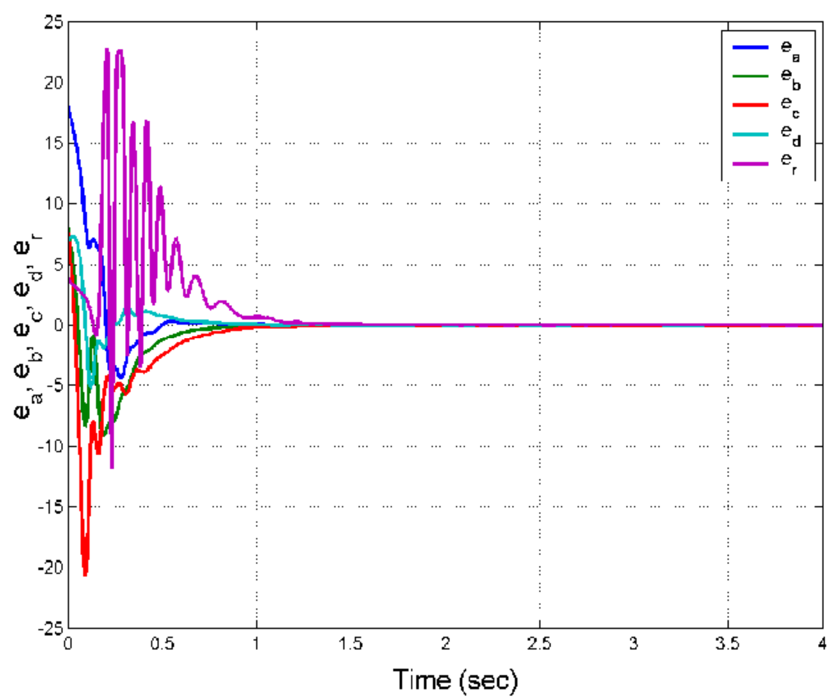

Figure 11. Time-History of the Parameter Estimation Errors e_a, e_b, e_c, e_d

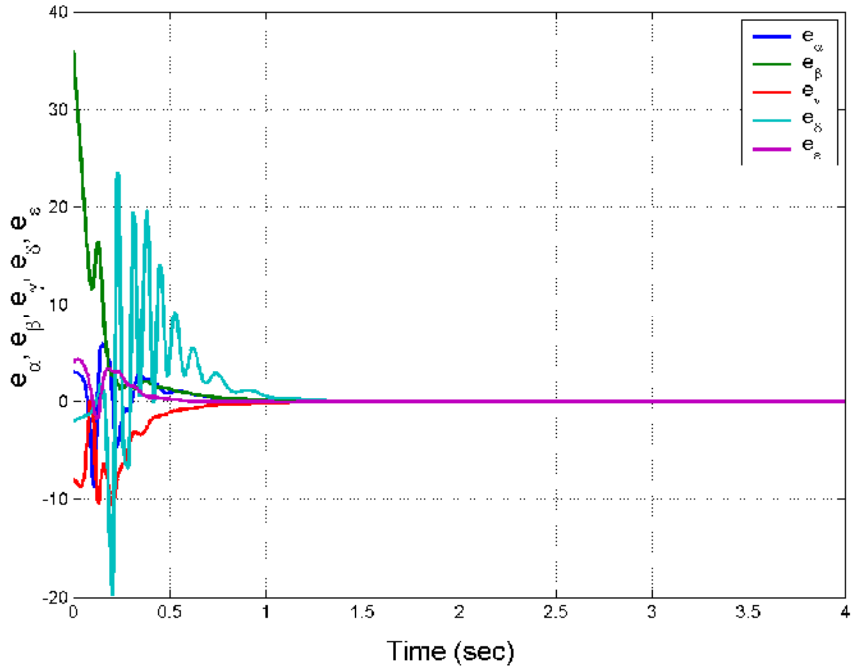

Figure 12. Time-History of the Parameter Estimation Errors $e_\alpha, e_\beta, e_\gamma, e_\delta, e_\varepsilon$

7. CONCLUSIONS

This paper derived new results for the active synchronizer design for achieving hybrid synchronization of hyperchaoticZhengsystems (2010) and hyperchaotic Yu systems (2012). Using Lyapunov control theory,adaptive control laws were derived for globally hybrid synchronizing the states of identical hyperchaotic Zheng systems, identical hyperchaotic Yu systems and non-identical hyperchaotic Zheng and Yu systems. Numerical simulations using MATLABwere shown to validate and illustrate the hybrid synchronization results for hyperchaotic Zheng and Yu systems.

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