ADAPTIVE CONTROLLER AND SYNCHRONIZER DESIGN FOR HYPERCHAOTIC ZHOU SYSTEM WITH UNKNOWN PARAMETERS

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ABSTRACT

In this paper, we establish new results for the adaptive controller and synchronizer design for the hyperchaotic Zhou system (2009), when the parameters of the system are unknown. Using adaptive control theory and Lyapunov stability theory, we first design an adaptive controller to stabilize the hyperchaotic Zhou system to its unstable equilibrium at the origin. Next, using adaptive control theory and Lyapunov stability theory, we design an adaptive controller to achieve global chaos synchronization of the identical hyperchaotic Zhou systems with unknown parameters. Simulations have been provided for adaptive controller and synchronizer designs to validate and illustrate the effectiveness of the schemes.

KEYWORDS

Adaptive Control, Adaptive Synchronization, Hyperchaos, Lyapunov Stability Theory.

1. INTRODUCTION

Hyperchaotic systems have been defined as chaotic systems having more than one positive Lyapunov exponent. Hyperchaotic systems exhibit complex dynamics and characteristics such as high capacity, high security and high efficiency. Some classical hyperchaotic systems are hyperchaotic Rössler system [1], hyperchaotic Lorenz-Haken system [2], hyperchaotic Chua's circuit [3], hyperchaotic Chen system [4], hyperchaotic Lü system [5], etc.

The control of a chaotic system is to devise a state feedback control law to stabilize the system around its unstable equilibrium points [6-7]. Active control [8] is applied when the system parameters are known and adaptive control [9] is applied when the system parameters are unknown.

Synchronization of chaotic systems is said to occur when a chaotic attractor drives another chaotic attractor. In the last two decades, there has been considerable interest devoted to the synchronization of chaotic and hyperchaotic systems.

In their seminal paper in 1990, Pecora and Carroll [10] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. Subsequently, chaos synchronization has been applied in a wide variety of fields including physics [11], chemistry [12], ecology [13], secure communications [14-15], cardiology [16], robotics [17], etc.

Some common methods applied to the chaos synchronization problem are active control method [18-22], adaptive control method [23-27], sampled-data feedback method [28], time-delay feedback method [29], backstepping method [30-33], sliding mode control method [34-38], etc.

This research paper has been organized as follows. In Section 2, we provide an analysis and description of the hyperchaotic Zhou system ([39], 2009). In Section 3, we detail our new results for the adaptive control of the hyperchaotic Zhou system with unknown parameters. In Section 4, we detail our new results for the adaptive synchronization of the identical hyperchaotic Zhou systems with unknown parameters. In Section 5, we give the conclusions of this research paper.

2. SYSTEM DESCRIPTION

The hyperchaotic Zhou system ([39], 2009) is described by the four-dimensional dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= cx_2 - x_1 x_3 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \\ \dot{x}_4 &= dx_1 + 0.5 x_2 x_3 \end{aligned} \tag{1}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, d are constant, positive parameters of the system.

The system (1) exhibits *hyperchaotic* behaviour when the parameter values are

$$a = 35, \quad b = 3, \quad c = 12, \quad 0 < d < 34.8 \tag{2}$$

Figure 1 depicts the hyperchaotic phase portrait of the hyperchaotic Zhou system (1), where, for simulation, the values of a, b, c are as given in (2) and the value of d is chosen as $d = 1.$

When the parameter values are taken as in (2) for the hyperchaotic system (1), the system linearization matrix at the equilibrium point $E_0 = (0, 0, 0, 0)$ is given by

$$A = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & c & 0 & 0 \\ 0 & 0 & -b & 0 \\ d & 0 & 0 & 0 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = c, \quad \lambda_2 = -b, \quad \lambda_3 = \frac{-a + \sqrt{a^2 + 4d}}{2}, \quad \lambda_4 = \frac{-a - \sqrt{a^2 + 4d}}{2}$$

Since λ_1 is a positive eigenvalues of A, it is immediate from Lyapunov stability theory [40] that the hyperchaotic Zhou system (1) is unstable at the equilibrium point $E_0 = (0, 0, 0, 0).$

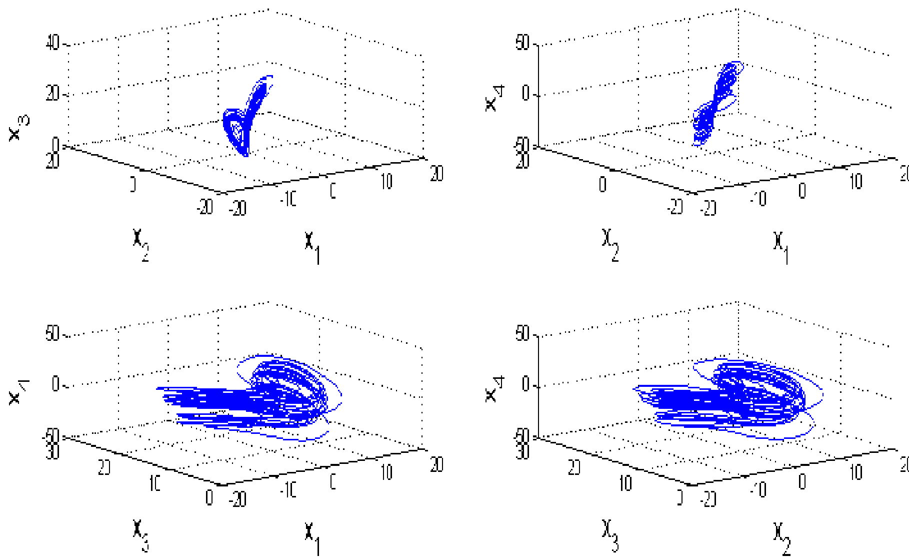

Figure 1. Phase Portrait of the Hyperchaotic Zhou System

3. ADAPTIVE CONTROL OF THE QI-CHEN CHAOTIC SYSTEM

3.1 Main Results

In this section, we design an adaptive controller for globally stabilizing the hyperchaotic Zhou system (2009) with unknown parameters.

Thus, we consider the controlled hyperchaotic Zhou system, which is described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 + u_1 \\
\dot{x}_2 &= cx_2 - x_1 x_3 + u_2 \\
\dot{x}_3 &= -bx_3 + x_1 x_2 + u_3 \\
\dot{x}_4 &= dx_1 + 0.5 x_2 x_3 + u_4
\end{aligned} \tag{3}$$

where u_1, u_2, u_3 and u_4 are feedback controllers to be designed using the states x_1, x_2, x_3, x_4 and estimates $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ of the unknown parameters a, b, c, d of the system.

In order to ensure that the controlled system (3) globally converges to the origin asymptotically, we consider the following adaptive control functions

$$\begin{aligned}
u_1(t) &= -\hat{a}(t)(x_2 - x_1) - x_4 - k_1 x_1 \\
u_2(t) &= -\hat{c}(t) x_2 + x_1 x_3 - k_2 x_2 \\
u_3(t) &= -x_1 x_2 + \hat{b}(t) x_3 - k_3 x_3 \\
u_4(t) &= -\hat{d}(t) x_1 - 0.5 x_2 x_3 - k_4 x_4
\end{aligned} \tag{4}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ are estimates of the parameters a, b, c, d, respectively, and $k_i, (i = 1, 2, 3, 4)$ are positive constants.

Substituting the control law (4) into the controlled hyperchaotic Zhou dynamics (3), we obtain

$$\begin{aligned} \dot{x}_1 &= (a - \hat{a})\ (x_2 - x_1) - k_1\ x_1 \\ \dot{x}_2 &= (c - \hat{c}) x_1 - k_2\ x_2 \\ \dot{x}_3 &= -(b - \hat{b})\ x_3 - k_3\ x_3 \\ \dot{x}_4 &= (d - \hat{d})\ x_1 - k_4\ x_4 \end{aligned} \tag{5}$$

We define the parameter estimation errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c}, \quad e_d = d - \hat{d} \tag{6}$$

Using (6), the closed-loop dynamics (5) can be expressed simply as

$$\begin{aligned} \dot{x}_1 &= e_a (x_2 - x_1) - k_1\ x_1 \\ \dot{x}_2 &= e_c x_1 - k_2\ x_2 \\ \dot{x}_3 &= -e_b\ x_3 - k_3\ x_3 \\ \dot{x}_4 &= e_d\ x_1 - k_4\ x_4 \end{aligned} \tag{7}$$

For deriving an update law for adjusting the parameter estimates, we apply the Lyapunov approach.

Consider the quadratic Lyapunov function

$$V(x_1, x_2, x_3, x_4, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 \right), \tag{8}$$

which is a positive definite function on R^8.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}}, \quad \dot{e}_d = -\dot{\hat{d}} \tag{9}$$

Differentiating V along the trajectories of (7) and using (9), we obtain

$$\begin{aligned} \dot{V} &= -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a \left[x_1 (x_2 - x_1) - \dot{\hat{a}} \right] + e_b \left(-x_3^2 - \dot{\hat{b}} \right) \\ &\quad + e_c \left(x_2^2 - \dot{\hat{c}} \right) + e_d \left(x_1 x_4 - \dot{\hat{d}} \right) \end{aligned} \tag{10}$$

In view of Eq. (10), the estimated parameters are updated by the following law:

$$\begin{aligned}
\dot{\hat{a}} &= x_1(x_2 - x_1) + k_5 e_a \\
\dot{\hat{b}} &= -x_3^2 + k_6 e_b \\
\dot{\hat{c}} &= x_2^2 + k_7 e_c \\
\dot{\hat{d}} &= x_1 x_4 + k_8 e_d
\end{aligned} \tag{11}$$

where k_5, k_6, k_7 and k_8 are positive constants.

Substituting (11) into (10), we get

$$\dot{V} = -k_1\, x_1^2 - k_2\, x_2^2 - k_3\, x_3^2 - k_4\, x_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 \tag{12}$$

which is a negative definite function on R^8.

Thus, by Lyapunov stability theory [40], we obtain the following result.

Theorem 1. *The controlled hyperchaotic Zhou system (1) having unknown system parameters is globally and exponentially stabilized for all initial conditions* $x(0) \in R^4$ *by the adaptive control law (4), where the parameter update law is given by (11) and the gains* k_i, $(i = 1, \ldots, 8)$ *are positive constants.* ■

3.2 Numerical Results

For numerical simulations, we have applied the fourth order Runge-Kutta method (MATLAB) with the step-size $h = 10^{-8}$ to solve the hyperchaotic Zhou system (3) with the adaptive control law (4) and the parameter update law (11). The parameters of the hyperchaotic Zhou system (3) are taken as

$$a = 35, \quad b = 3, \quad c = 12, \quad d = 1$$

For the adaptive and update laws, we take $k_i = 5, \quad (i = 1, 2, \ldots, 8)$.

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 9, \quad \hat{b}(0) = 12, \quad \hat{c}(0) = 4, \quad \hat{d}(0) = 6$$

The initial state of the controlled Qi-Chen system (3) is taken as

$$x_1(0) = 25, \quad x_2(0) = -16, \quad x_3(0) = 20, \quad x_4(0) = -30$$

When the adaptive control law (4) and the parameter update law (11) are used, the state trajectories of the controlled modified hyperchaotic Zhou system converge exponentially to the equilibrium $E_0 = (0, 0, 0, 0)$ as shown in Figure 2. The time-history of the parameter estimates is shown in Figure 3. The time-history of the parameter estimation errors is shown in Figure 4.

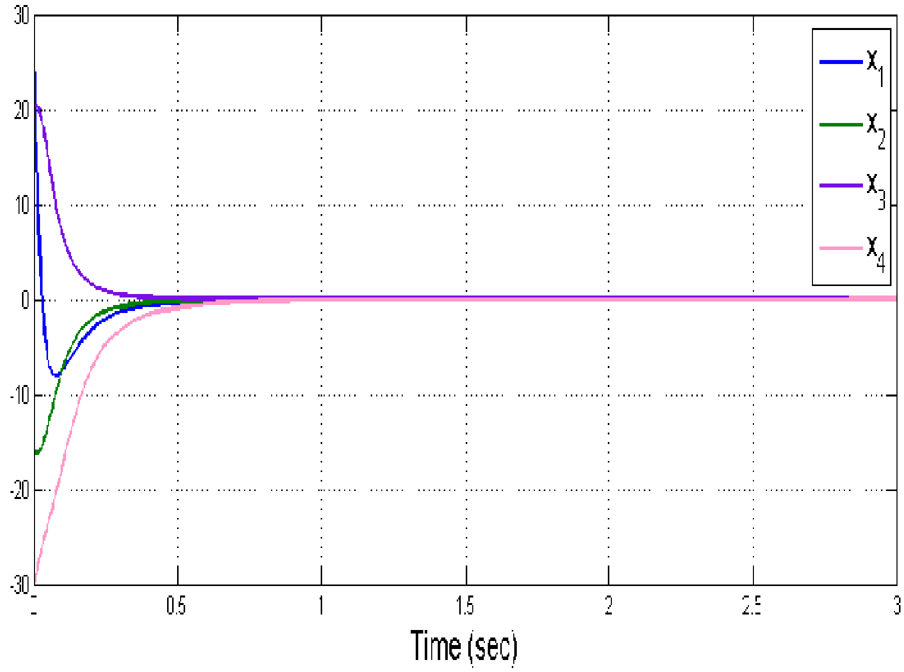

Figure 2. Time Responses of the Controlled Hyperchaotic Zhou System

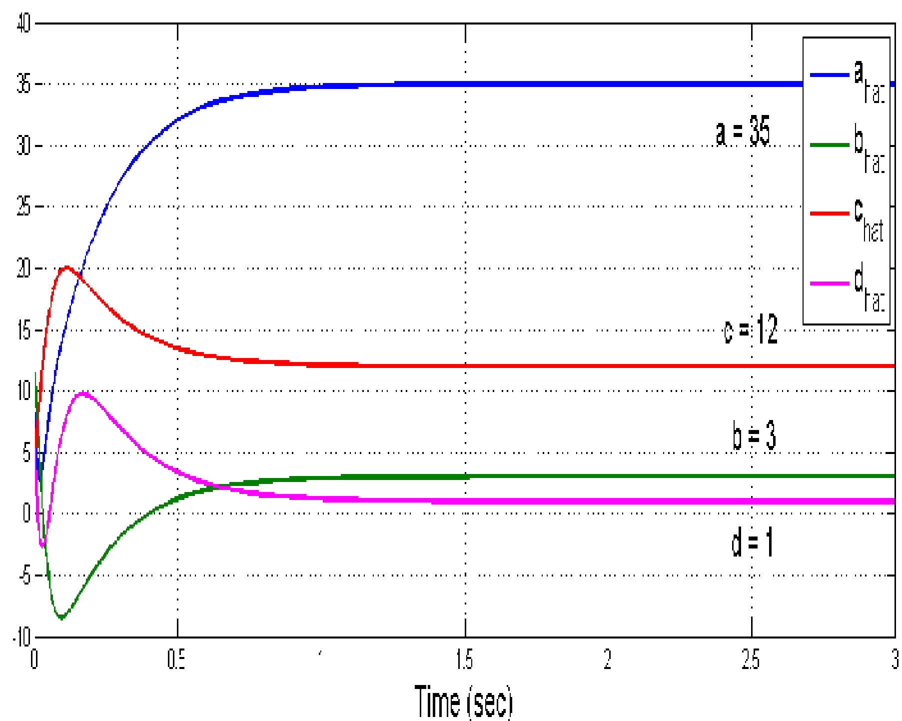

Figure 3. Time-History of the Parameter Estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{d}(t)$

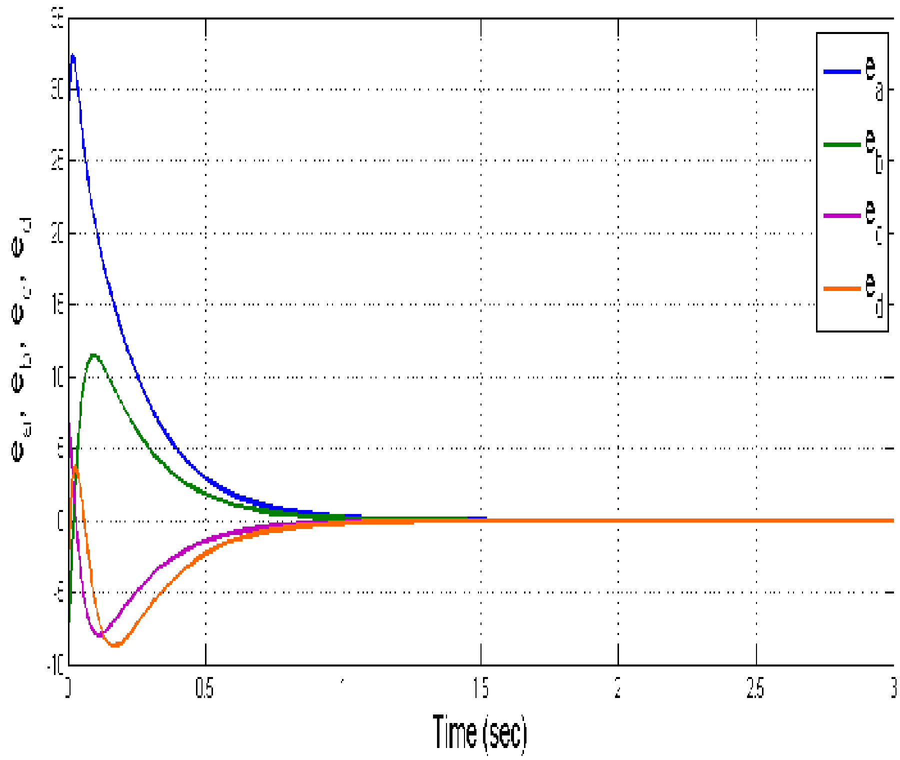

Figure 4. Time-History of the Parameter Estimates e_a, e_b, e_c, e_d

4. Adaptive Synchronization of Identical Hyperchaotic Zhou Systems

4.1 Theoretical Results

In this section, we derive new results for the adaptive synchronization of identical hyperchaotic Zhou systems (2009) with unknown parameters.

As the master system, we take the hyperchaotic Zhou dynamics described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= cx_2 - x_1 x_3 \\
\dot{x}_3 &= -bx_3 + x_1 x_2 \\
\dot{x}_4 &= dx_1 + 0.5 x_2 x_3
\end{aligned} \tag{13}$$

where $x_i,\ (i = 1, 2, 3, 4)$ are the state variables and a, b, c, d are unknown system parameters.

The system (13) is hyperchaotic when the parameter values are taken as

$$a = 35, \quad b = 3, \quad c = 12, \quad d = 1$$

As the slave system, we consider the modified hyperchaotic Zhou dynamics described by

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1\\ \dot{y}_2 &= cy_2 - y_1 y_3 + u_2\\ \dot{y}_3 &= -by_3 + y_1 y_2 + u_3\\ \dot{y}_4 &= dy_1 + 0.5 y_2 y_3 + u_4\end{aligned} \tag{14}$$

where y_i, $(i = 1, 2, 3, 4)$ are the state variables and u_i, $(i = 1, 2, 3, 4)$ are the nonlinear controllers to be designed.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \tag{15}$$

Then the error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1\\ \dot{e}_2 &= ce_2 - y_1 y_3 + x_1 x_3 + u_2\\ \dot{e}_3 &= -be_3 + y_1 y_2 - x_1 x_2 + u_3\\ \dot{e}_4 &= de_1 + 0.5(y_2 y_3 - x_2 x_3) + u_4\end{aligned} \tag{16}$$

Let us now define the adaptive control functions $u_1(t), u_2(t), u_3(t), u_4(t)$ as

$$\begin{aligned}u_1(t) &= -\hat{a}(t)(e_2 - e_1) - e_4 - k_1 e_1\\ u_2(t) &= -\hat{c}(t)e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2\\ u_3(t) &= \hat{b}(t)e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3\\ u_4(t) &= -\hat{d}(t)e_1 - 0.5(y_2 y_3 - x_2 x_3) - k_4 e_4\end{aligned} \tag{17}$$

where $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t)$ are estimates of the parameters a, b, c, d, respectively, and $k_i, (i = 1, 2, 3, 4)$ are positive constants.

Substituting the control law (17) into (16), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= (a - \hat{a})(e_2 - e_1) - k_1 e_1\\ \dot{e}_2 &= (c - \hat{c})e_2 - k_2 e_2\\ \dot{e}_3 &= -(b - \hat{b})e_3 - k_3 e_3\\ \dot{e}_4 &= (d - \hat{d})e_1 - k_4 e_4\end{aligned} \tag{18}$$

We define parameter estimation errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c}, \quad e_d = d - \hat{d} \tag{19}$$

Substituting (19) into (18), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e\ e_2 - k_2 e_2 \\ \dot{e}_3 &= -e_b e_3 - k_3 e_3 \\ \dot{e}_4 &= e_d e_1 - k_4 e_4 \end{aligned} \tag{20}$$

For the derivation of the update law for adjusting the parameter estimates, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V(e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{21}$$

which is a positive definite function on R^8.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}}, \quad \dot{e}_d = -\dot{\hat{d}} \tag{22}$$

Differentiating V along the trajectories of (20) and using (22), we obtain

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_b\left[-e_3^2 - \dot{\hat{b}}\right] + e_c\left[e_2^2 - \dot{\hat{c}}\right] \\ &+ e_d\left[e_1 e_4 - \dot{\hat{d}}\right] \end{aligned} \tag{23}$$

In view of Eq. (23), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1) + k_5 e_a \\ \dot{\hat{b}} &= -e_3^2 + k_6 e_b \\ \dot{\hat{c}} &= e_2^2 + k_7 e_c \\ \dot{\hat{d}} &= e_1 e_4 + k_8 e_d \end{aligned} \tag{24}$$

where k_5, k_6, k_7, k_8 are positive constants.

Substituting (24) into (23), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 \tag{25}$$

From (25), we find that $\dot{V}$ is a negative definite function on R^8.

Thus, by Lyapunov stability theory [40], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions.

Hence, we have proved the following result.

Theorem 2. The identical hyperchaotic Zhou systems (13) and (14) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (17), where the update law for parameters is given by (24) and $k_i, (i = 1, \ldots, 8)$ are positive constants. ■

4.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (13) and (14) with the adaptive control law (17) and the parameter update law (24).

We take the parameter values as in the hyperchaotic case, viz.

$$a = 35, \quad b = 3, \quad c = 12, \quad d = 1$$

We take the positive constants $k_i, \ (i = 1, \ldots, 8)$ as

$$k_i = 5 \quad \text{for} \quad i = 1, 2, \ldots, 8.$$

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 6, \quad \hat{b}(0) = 10, \quad \hat{c}(0) = 20, \quad \hat{d}(0) = 15$$

We take the initial values of the master system (13) as

$$x_1(0) = 7, \quad x_2(0) = -5, \quad x_3(0) = 16, \quad x_4(0) = -12$$

We take the initial values of the slave system (14) as

$$y_1(0) = 14, \quad y_2(0) = 28, \quad y_3(0) = -10, \quad y_4(0) = 6$$

Figure 5 shows the adaptive chaos synchronization of the identical hyperchaotic Zhou systems.

Figure 6 shows the time-history of the synchronization error e_1, e_2, e_3, e_4.

Figure 7 shows the time-history of the parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$.

From this figure, it is clear that the parameter estimates converge to the original values $a = 35, \ b = 3, \ c = 12, \ d = 1$, respectively.

Figure 8 shows the time-history of the parameter estimation errors e_a, e_b, e_c, e_d.

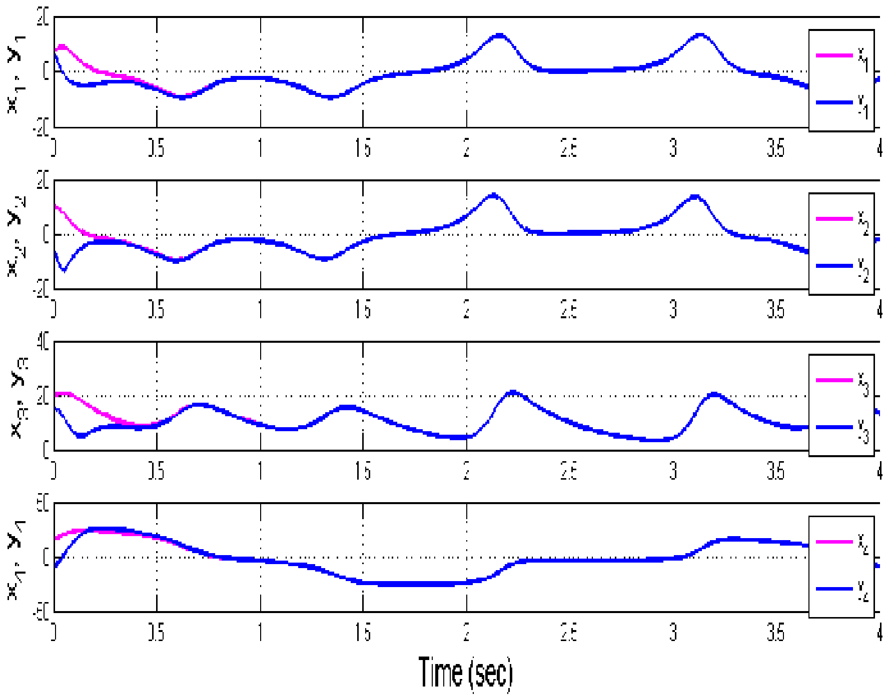

Figure 5. Adaptive Synchronization of the hyperchaotic Zhou Systems

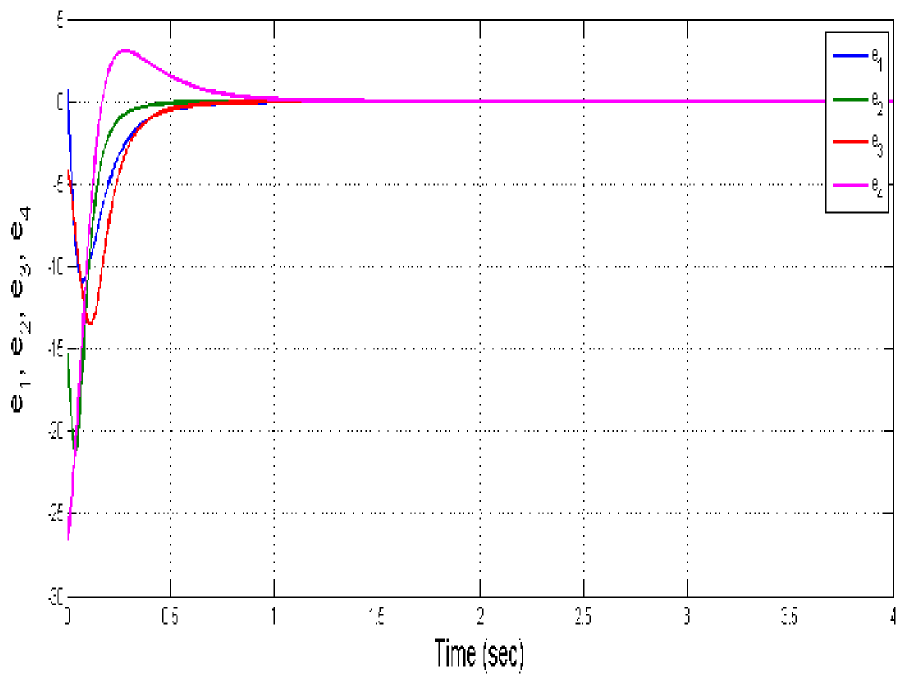

Figure 6. Time-History of the Synchronization Error e_1, e_2, e_3, e_4

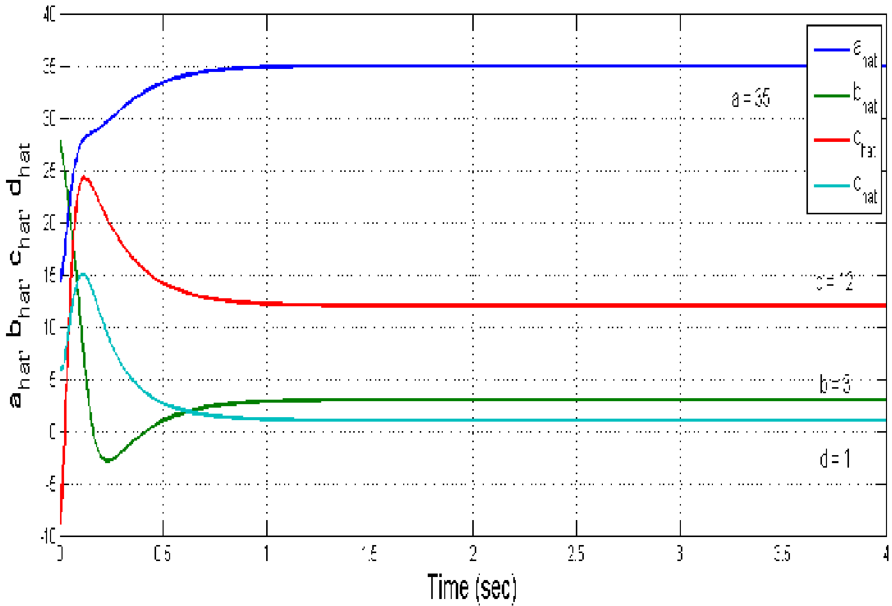

Figure 7. Time-History of the Parameter Estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t)$

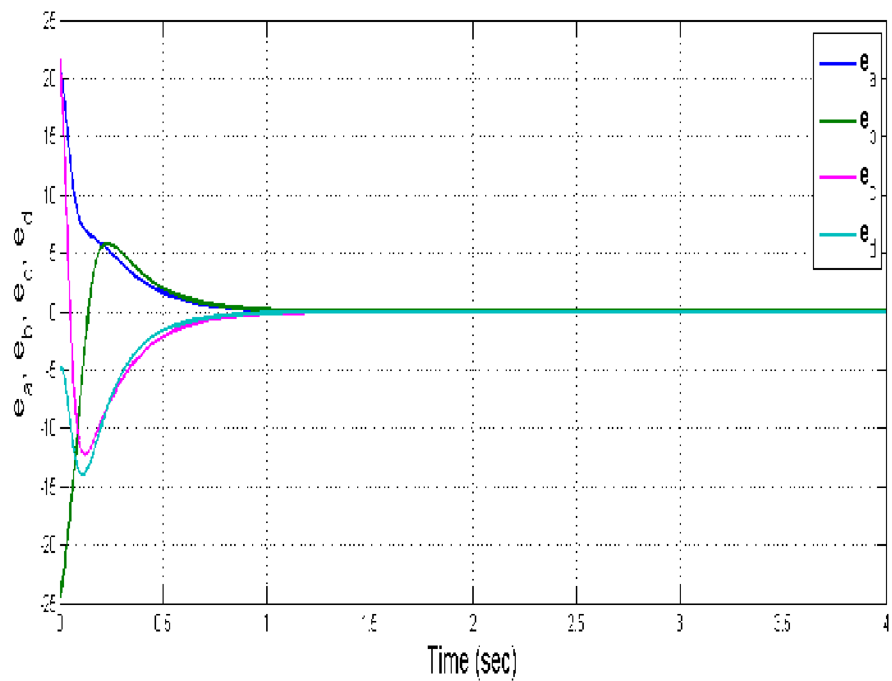

Figure 8. Time-History of the Parameter Estimation Error e_a, e_b, e_c, e_d

5. CONCLUSIONS

In this paper, we derived new results for the adaptive stabilization and synchronization of the hyperchaotic Zhou system (2009) with unknown system parameters. First, an adaptive controller law was designed for stabilizing the hyperchaotic Zhou system (2009) to its unstable equilibrium at the origin. Next, an adaptive synchronizer law was designed for synchronizing identical hyperchaotic Zhou systems. The main results of this paper on adaptive control and adaptive synchronization were established using adaptive control theory and Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient to achieve control and synchronization of the hyperchaotic Zhou system. Numerical simulations have been provided to validate and demonstrate the effectiveness of the proposed adaptive control and synchronization schemes.

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