ACTIVE CONTROLLER DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC ZHENG AND HYPERCHAOTIC YU SYSTEMS

Sundarapandian Vaidyanathan[1]

[1]Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi, Chennai-600 062, Tamil Nadu, INDIA
sundarvtu@gmail.com

ABSTRACT

This paper deals with a new research problem in the chaos literature, viz. hybrid synchronization of a pair of chaotic systems called the master and slave systems. In the hybrid synchronization design of master and slave systems, one part of the systems, viz. their odd states, are completely synchronized (CS), while the other part, viz. their even states, are completely anti-synchronized (AS) so that CS and AS co-exist in the process of synchronization. This research work deals with the hybrid synchronization of hyperchaotic Zheng systems (2010) and hyperchaotic Yu systems (2012). The main results of this hybrid synchronization research work have been proved using Lyapunov stability theory. Numerical examples of the hybrid synchronization results are shown along with MATLAB simulations for the hyperchaotic Zheng and hyperchaotic Yu systems.

KEYWORDS

Hybrid Synchronization, Active Control, Chaos, Hyperchaos, Hyperchaotic Systems.

1. INTRODUCTION

Hyperchaotic systems are typically defined as chaotic systems possessing two or more positive Lyapunov exponents. These systems have several miscellaneous applications in Engineering and Science. The first known hyperchaotic system was discovered by O.E. Rössler ([1], 1979).

Hyperchaotic systems have many useful features like high security, high capacity and high efficiency. Hence, the hyperchaotic systems have important applications in areas like neural networks [2], oscillators [3], communication [4-5], encryption [6], synchronization [7], etc.

For the synchronization of chaotic systems, there are many methods available in the chaos literature like OGY method [8], PC method [9], backstepping method [10-12], sliding control method [13-15], active control method [16-18], adaptive control method [19-20], sampled-data feedback control method [21], time-delay feedback method [22], etc.

In the hybrid synchronization of a pair of chaotic systems called the master and slave systems, one part of the systems, viz. the odd states, are completely synchronized (CS), while the other part of the systems, viz. the even states, are anti-synchronized so that CS and AS co-exist in the process of synchronization of the two systems.

This paper focuses upon active controller design for the hybrid synchronization of hyperchaotic Zheng systems ([23], 2010) and hyperchaotic Yu systems ([24], 2012). The main results derived in this paper have been proved using stability theorems of Lyapunov stability theory [25].

2. HYBRID SYNCHRONIZATION PROBLEM

The *master system* is described by the chaotic dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part.

The *slave system* is described by the chaotic dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part and $u \in R^n$ is the active controller to be designed.

For the pair of chaotic systems (1) and (2), the *hybrid synchronization error* is defined as

$$e_i = \begin{cases} y_i - x_i, & \text{if } i \text{ is odd} \\ y_i + x_i, & \text{if } i \text{ is even} \end{cases} \tag{3}$$

The error dynamics is obtained as

$$\dot{e}_i = \begin{cases} \sum_{j=1}^{n} (b_{ij} y_j - a_{ij} x_j) + g_i(y) - f_i(x) + u_i & \text{if } i \text{ is odd} \\ \sum_{j=1}^{n} (b_{ij} y_j + a_{ij} x_j) + g_i(y) + f_i(x) + u_i & \text{if } i \text{ is even} \end{cases} \tag{4}$$

The design goal is to find a feedback controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \text{ for all } e(0) \in \boldsymbol{R^n} \tag{5}$$

Using the matrix method, we consider a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite matrix. It is noted that $V : R^n \to R$ is a positive definite function.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [25], the error dynamics (4) is globally exponentially stable. Hence, the states of the chaotic systems (1) and (2) will be globally and exponentially hybrid synchronized for all initial conditions $x(0), y(0) \in R^n$.

3. HYPERCHAOTIC SYSTEMS

The hyperchaotic Zheng system ([23], 2010) has the 4-D dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= bx_1 + cx_2 + x_4 + x_1x_3 \\ \dot{x}_3 &= -x_1^2 - rx_3 \\ \dot{x}_4 &= -dx_2\end{aligned} \quad (8)$$

where a, b, c, r, d are constant, positive parameters of the system.

The Zheng system (8) exhibits a hyperchaotic attractor for the parametric values

$$a = 20, \quad b = 14, \quad c = 10.6, \quad d = 4, \quad r = 2.8 \quad (9)$$

The Lyapunov exponents of the system (8) for the parametric values in (9) are

$$L_1 = 1.8892, \quad L_2 = 0.2268, \quad L_3 = 0, \quad L_4 = -14.3130 \quad (10)$$

Since there are two positive Lyapunov exponents in (10), the Zheng system (8) is hyperchaotic for the parametric values (9).

The strange attractor of the hyperchaotic Zheng system is depicted in Figure 1.

The hyperchaotic Yu system ([24], 2012) has the 4-D dynamics

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \beta x_1 - x_1x_3 + \gamma x_2 + x_4 \\ \dot{x}_3 &= -\delta x_3 + e^{x_1x_2} \\ \dot{x}_4 &= -\varepsilon x_1\end{aligned} \quad (11)$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are constant, positive parameters of the system.

The Yu system (11) exhibits a hyperchaotic attractor for the parametric values

$$\alpha = 10, \quad \beta = 40, \quad \gamma = 1, \quad \delta = 3, \quad \varepsilon = 8 \quad (12)$$

The Lyapunov exponents of the system (11) for the parametric values in (12) are

$$L_1 = 1.6877, \quad L_2 = 0.1214, \quad L_3 = 0, \quad L_4 = -13.7271 \quad (13)$$

Since there are two positive Lyapunov exponents in (13), the Yu system (11) is hyperchaotic for the parametric values (12).

The strange attractor of the hyperchaotic Yu system is displayed in Figure 2.

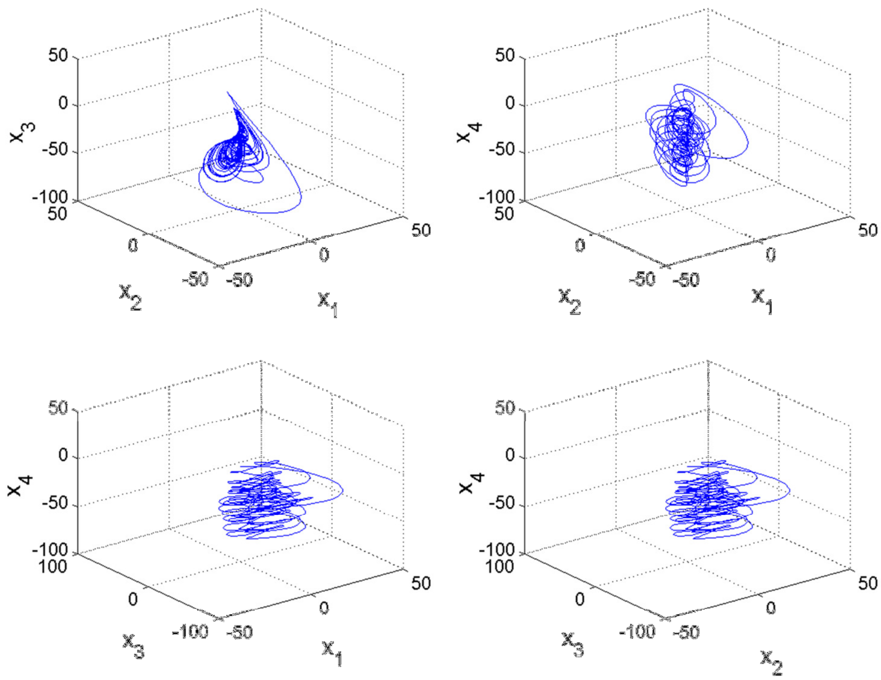

Figure 1. The Strange Attractor of the Hyperchaotic Zheng System

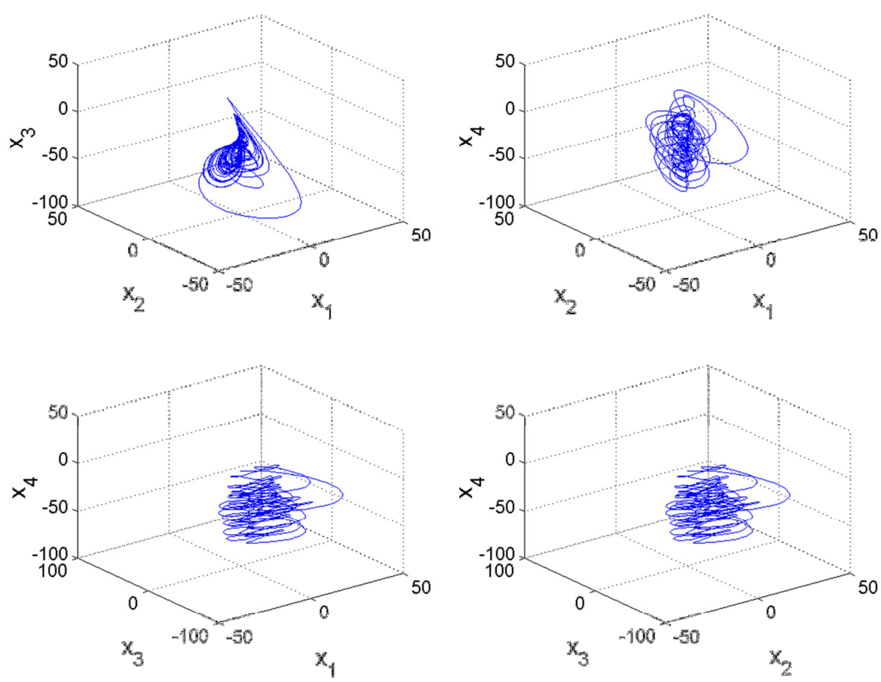

Figure 2. The Strange Attractor of the Hyperchaotic Yu System

4. ACTIVE CONTROL DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC ZHENG SYSTEMS

In this section, we design an active controller for the hybrid synchronization of two identical hyperchaotic Zheng systems (2010) and prove our main result using Lyapunov stability theory.

The hyperchaotic Zheng system is taken as the master system, whose dynamics is given by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= bx_1 + cx_2 + x_4 + x_1x_3 \\
\dot{x}_3 &= -x_1^2 - rx_3 \\
\dot{x}_4 &= -dx_2
\end{aligned} \tag{14}$$

where a, b, c, d, r are positive parameters of the system and $x \in R^4$ is the state of the system.

The hyperchaotic Zheng system is also taken as the slave system, whose dynamics is given by

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= by_1 + cy_2 + y_4 + y_1y_3 + u_2 \\
\dot{y}_3 &= -y_1^2 - ry_3 + u_3 \\
\dot{y}_4 &= -dy_2 + u_4
\end{aligned} \tag{15}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.

For the hybrid synchronization, the error e is defined as

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{16}$$

A simple calculation using the dynamics (14) and (15) yields the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 - 2ax_2 - 2x_4 + u_1 \\
\dot{e}_2 &= be_1 + ce_2 + e_4 + 2bx_1 + y_1y_3 + x_1x_3 + u_2 \\
\dot{e}_3 &= -re_3 - y_1^2 + x_1^2 + u_3 \\
\dot{e}_4 &= -de_2 + u_4
\end{aligned} \tag{17}$$

We choose the active controller for achieving hybrid synchronization as

$$\begin{aligned} u_1 &= -a(e_2 - e_1) - e_4 + 2ax_2 + 2x_4 - k_1e_1 \\ u_2 &= -be_1 - ce_2 - e_4 - 2bx_1 - y_1y_3 - x_1x_3 - k_2e_2 \\ u_3 &= re_3 + y_1^2 - x_1^2 - k_3e_3 \\ u_4 &= de_2 - k_4e_4 \end{aligned} \tag{18}$$

where k_i, $(i = 1, 2, 3, 4)$ are positive gains.

Substituting (18) into (17), the error dynamics simplifies into

$$\begin{aligned} \dot{e}_1 &= -k_1e_1 \\ \dot{e}_2 &= -k_2e_2 \\ \dot{e}_3 &= -k_3e_3 \\ \dot{e}_4 &= -k_4e_4 \end{aligned} \tag{19}$$

Thus, we get the following result.

Theorem 4.1 The active control law defined by Eq. (18) achieves global and exponential hybrid synchronization of the identical hyperchaotic Zheng systems (14) and (15) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The result is proved using Lyapunov stability theory [25] for global exponential stability.

We take the quadratic Lyapunov function

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{20}$$

which is a positive definite function on R^4.

When we differentiate (18) along the trajectories of (17), we get

$$\dot{V}(e) = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 \tag{21}$$

which is a negative definite function on R^4.

Hence, the error dynamics (19) is globally exponentially stable for all $e(0) \in R^4$.

This completes the proof. ■

Next, we illustrate our hybrid synchronization results with MATLAB simulations.

The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Zheng systems (14) and (15) with the active nonlinear controller (18).

The feedback gains in the active controller (18) are taken as $k_i = 5, \ (i = 1, 2, 3, 4)$.

The parameters of the hyperchaotic Zheng systems are taken as in the hyperchaotic case, *i.e.*

$$a = 20, \quad b = 14, \quad c = 10.6, \quad d = 4, \quad r = 2.8$$

For simulations, the initial conditions of the hyperchaotic Zheng system (14) are chosen as

$$x_1(0) = -14, \quad x_2(0) = 7, \quad x_3(0) = -5, \quad x_4(0) = 23$$

Also, the initial conditions of the hyperchaotic Zheng system (15) are chosen as

$$y_1(0) = 8, \quad y_2(0) = -21, \quad y_3(0) = 10, \quad y_4(0) = -27$$

Figure 3 depicts the hybrid synchronization of the identical hyperchaotic Zheng systems.

Figure 4 depicts the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

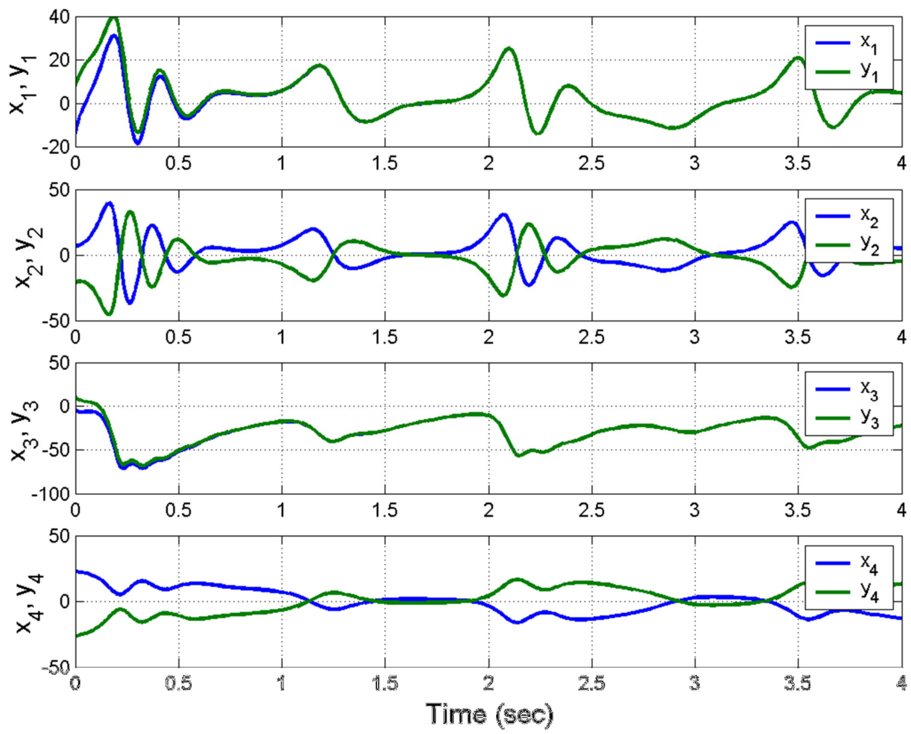

Figure 3. Hybrid Synchronization of Identical Hyperchaotic Zheng Systems

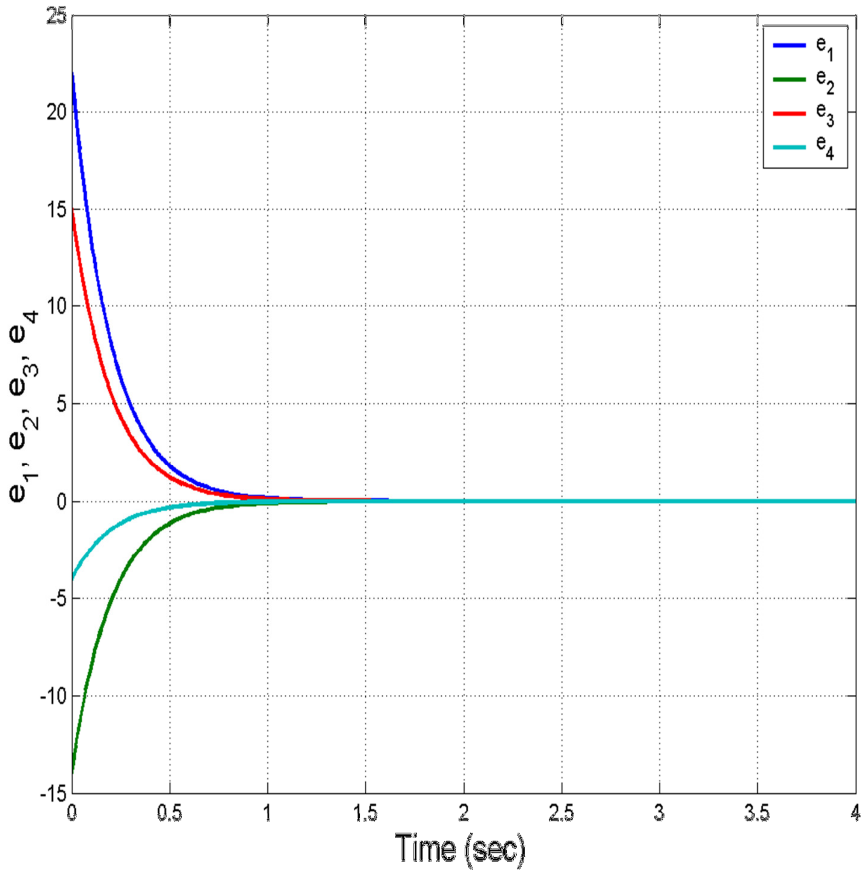

Figure 4. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

5. Active Controller Design for the Hybrid Synchronization Design of Hyperchaotic Yu Systems

In this section, we design an active controller for the hybrid synchronization of two identical hyperchaotic Yu systems (2012) and prove our main result using Lyapunov stability theory.

The hyperchaotic Yu system is taken as the master system, whose dynamics is given by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \beta x_1 - x_1 x_3 + \gamma x_2 + x_4 \\ \dot{x}_3 &= -\delta x_3 + e^{x_1 x_2} \\ \dot{x}_4 &= -\varepsilon x_1 \end{aligned} \tag{22}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are positive parameters of the system and $x \in R^4$ is the state of the system.

The hyperchaotic Yu system is taken as the slave system, whose dynamics is given by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \beta y_1 - y_1 x_3 + \gamma y_2 + y_4 + u_2 \\ \dot{y}_3 &= -\delta y_3 + e^{y_1 y_2} + u_3 \\ \dot{y}_4 &= -\varepsilon y_1 + u_4\end{aligned} \tag{23}$$

where $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.

For the hybrid synchronization, the error e is defined as

$$\begin{aligned}e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4\end{aligned} \tag{24}$$

We obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) - 2\alpha x_2 + u_1 \\ \dot{e}_2 &= \beta e_1 + \gamma e_2 + e_4 + 2\beta x_1 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -\delta e_3 + e^{y_1 y_2} - e^{x_1 x_2} + u_3 \\ \dot{e}_4 &= -\varepsilon e_1 - 2\varepsilon x_1 + u_4\end{aligned} \tag{25}$$

We choose the active controller for achieving hybrid synchronization as

$$\begin{aligned}u_1 &= -\alpha(e_2 - e_1) + 2\alpha x_2 - k_1 e_1 \\ u_2 &= -\beta e_1 - \gamma e_2 - e_4 - 2\beta x_1 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 &= \delta e_3 - e^{y_1 y_2} + e^{x_1 x_2} - k_3 e_3 \\ u_4 &= \varepsilon e_1 + 2\varepsilon x_1 - k_4 e_4\end{aligned} \tag{26}$$

where k_i, $(i = 1, 2, 3, 4)$ are positive gains.

By the substitution of (26) into (25), the error dynamics is simplified as

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4\end{aligned} \tag{27}$$

Thus, we obtain the following result.

Theorem 5.1 The active control law defined by Eq. (26) achieves global and exponential hybrid synchronization of the identical hyperchaotic Yu systems (22) and (23) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The result is proved using Lyapunov stability theory [25] for global exponential stability. We take the quadratic Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{28}$$

which is a positive definite function on R^4.

When we differentiate (26) along the trajectories of (25), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{29}$$

which is a negative definite function on R^4.

Hence, the error dynamics (27) is globally exponentially stable for all $e(0) \in R^4$.

This completes the proof. ■

Next, we illustrate our hybrid synchronization results with MATLAB simulations.

The classical fourth-order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic Yu systems (22) and (23) with the active controller defined by (26).

The feedback gains in the active controller (26) are taken as

$$k_i = 5, \ (i = 1, 2, 3, 4).$$

The parameters of the hyperchaotic Yu systems are taken as in the hyperchaotic case, *i.e.*

$$\alpha = 10, \ \ \beta = 40, \ \ \gamma = 1, \ \ \delta = 3, \ \ \varepsilon = 8$$

For simulations, the initial conditions of the hyperchaotic Yu system (22) are chosen as

$$x_1(0) = 7, \ \ x_2(0) = -2, \ \ x_3(0) = 6, \ \ x_4(0) = 1$$

Also, the initial conditions of the hyperchaotic Yu system (23) are chosen as

$$y_1(0) = 5, \ \ y_2(0) = 4, \ \ y_3(0) = 1, \ \ y_4(0) = 8$$

Figure 5 depicts the hybrid synchronization of the identical hyperchaotic Yu systems.

Figure 6 depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4.

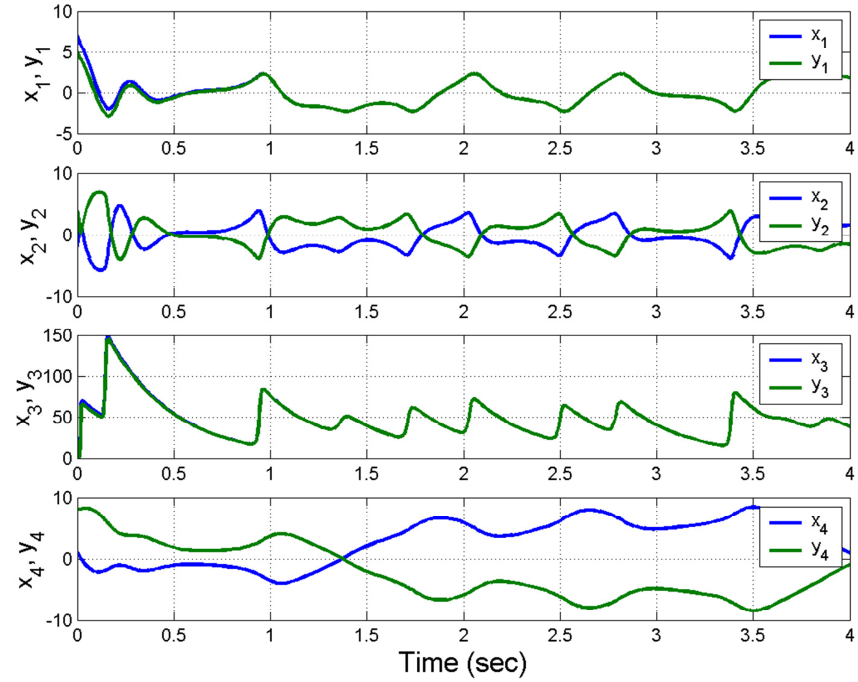

Figure 5. Hybrid Synchronization of Identical Hyperchaotic Yu Systems

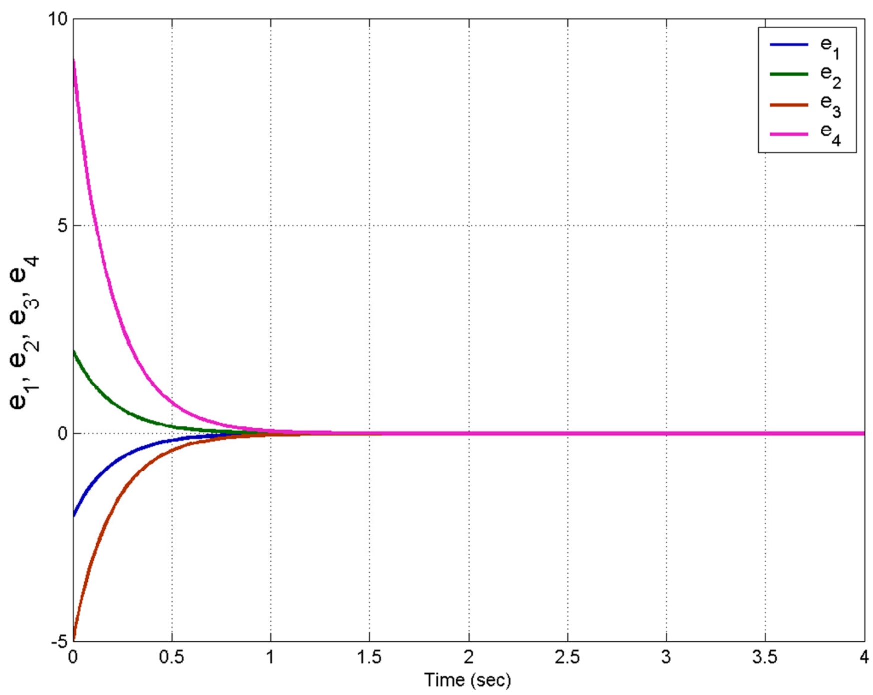

Figure 6. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

6. ACTIVE CONTROLLER DESIGN FOR THE HYBRID SYNCHRONIZATION OF HYPERCHAOTIC ZHENG AND HYPERCHAOTIC YU SYSTEMS

In this section, we design an active controller for the hybrid synchronization of hyperchaotic Zheng system (2010) and hyperchaotic Yu system (2012) and establish our main result using Lyapunov stability theory.

The hyperchaotic Zheng system is taken as the master system, whose dynamics is given by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= bx_1 + cx_2 + x_4 + x_1 x_3 \\ \dot{x}_3 &= -x_1^2 - rx_3 \\ \dot{x}_4 &= -dx_2 \end{aligned} \tag{30}$$

where a, b, c, d, r are positive parameters of the system and $x \in R^4$ is the state of the system.

The hyperchaotic Yu system is taken as the slave system, whose dynamics is given by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \beta y_1 - y_1 x_3 + \gamma y_2 + y_4 + u_2 \\ \dot{y}_3 &= -\delta y_3 + e^{y_1 y_2} + u_3 \\ \dot{y}_4 &= -\varepsilon y_1 + u_4 \end{aligned} \tag{31}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon$ are positive parameters of the system, $y \in R^4$ is the state and u_1, u_2, u_3, u_4 are the active controllers to be designed.

For the hybrid synchronization, the error e is defined as

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4 \end{aligned} \tag{32}$$

We obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= \alpha(y_2 - y_1) - a(x_2 - x_1) - x_4 + u_1 \\ \dot{e}_2 &= \beta y_1 + bx_1 + \gamma y_2 + cx_2 + e_4 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\delta y_3 + rx_3 + e^{y_1 y_2} + x_1^2 + u_3 \\ \dot{e}_4 &= -\varepsilon y_1 - dx_2 + u_4 \end{aligned} \tag{33}$$

We choose the active controller for achieving hybrid synchronization as

$$\begin{aligned}
u_1 &= -\alpha(y_2 - y_1) + a(x_2 - x_1) + x_4 - k_1 e_1 \\
u_2 &= -\beta y_1 - b x_1 - \gamma y_2 - c x_2 - e_4 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\
u_3 &= \delta y_3 - r x_3 - e^{y_1 y_2} - x_1^2 - k_3 e_3 \\
u_4 &= \varepsilon y_1 + d x_2 - k_4 e_4
\end{aligned} \tag{34}$$

where $k_i, \ (i = 1, 2, 3, 4)$ are positive gains.

By the substitution of (34) into (33), the error dynamics is simplified as

$$\begin{aligned}
\dot{e}_1 &= -k_1 e_1 \\
\dot{e}_2 &= -k_2 e_2 \\
\dot{e}_3 &= -k_3 e_3 \\
\dot{e}_4 &= -k_4 e_4
\end{aligned} \tag{35}$$

Thus, we obtain the following result.

Theorem 6.1 The active control law defined by Eq. (33) achieves global and exponential hybrid synchronization of the hyperchaotic Zheng system (30) and hyperchaotic Yu system (31) for all initial conditions $x(0), y(0) \in R^4$.

Proof. The proof is via Lyapunov stability theory [25] for global exponential stability.

We take the quadratic Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{36}$$

which is a positive definite function on R^4.

When we differentiate (34) along the trajectories of (33), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{37}$$

which is a negative definite function on R^4.

Hence, the error dynamics (35) is globally exponentially stable for all $e(0) \in R^4$.

This completes the proof. ■

Next, we illustrate our hybrid synchronization results with MATLAB simulations.

The classical fourth order Runge-Kutta method with time-step $h = 10^{-8}$ has been applied to solve the hyperchaotic systems (30) and (31) with the active controller defined by (34).

The feedback gains in the active controller (34) are taken as $k_i = 5, \ (i = 1,2,3,4).$

The parameters of the hyperchaotic Zheng and hyperchaotic Yu systems are taken as in the hyperchaotic case, *i.e.*

$$a = 20, \ b = 14, \ c = 10.6, \ d = 4, \ r = 2.8, \ \alpha = 10, \ \beta = 40, \ \gamma = 1, \ \delta = 3, \ \varepsilon = 8$$

For simulations, the initial conditions of the hyperchaotic Xu system (30) are chosen as

$$x_1(0) = 7, \ \ x_2(0) = -4, \ \ x_3(0) = -10, \ \ x_4(0) = 8$$

Also, the initial conditions of the hyperchaotic Li system (31) are chosen as

$$y_1(0) = 1, \ \ y_2(0) = 7, \ \ y_3(0) = -24, \ \ y_4(0) = 15$$

Figure 7 depicts the hybrid synchronization of the non-identical hyperchaotic Zheng and hyperchaotic Yu systems.

Figure 8 depicts the time-history of the hybrid synchronization errors e_1, e_2, e_3, e_4.

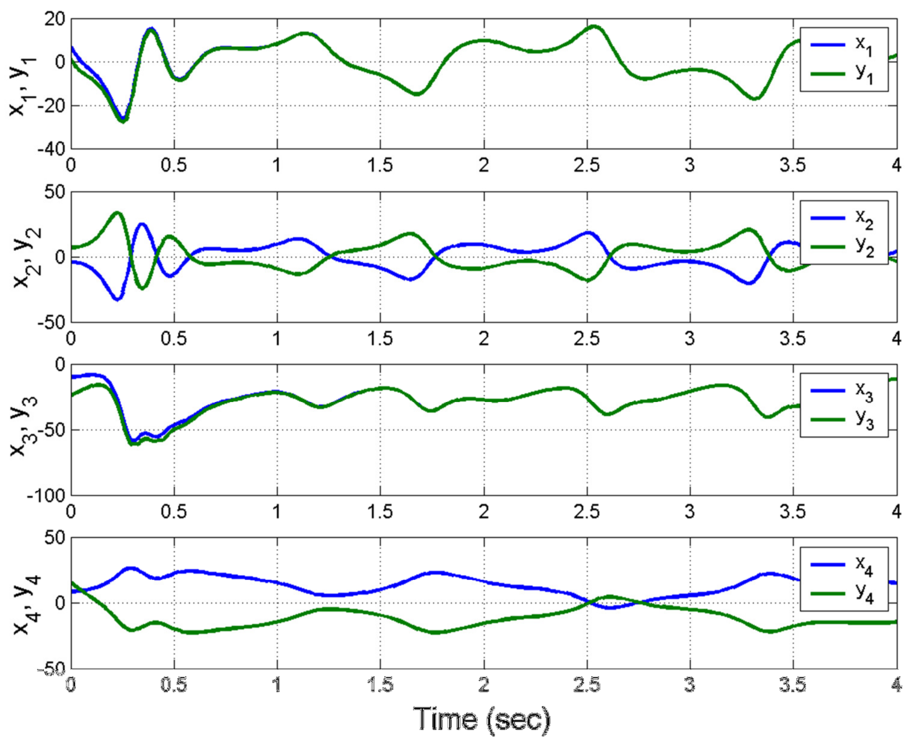

Figure 7. Hybrid Synchronization of Hyperchaotic Zheng and Yu Systems

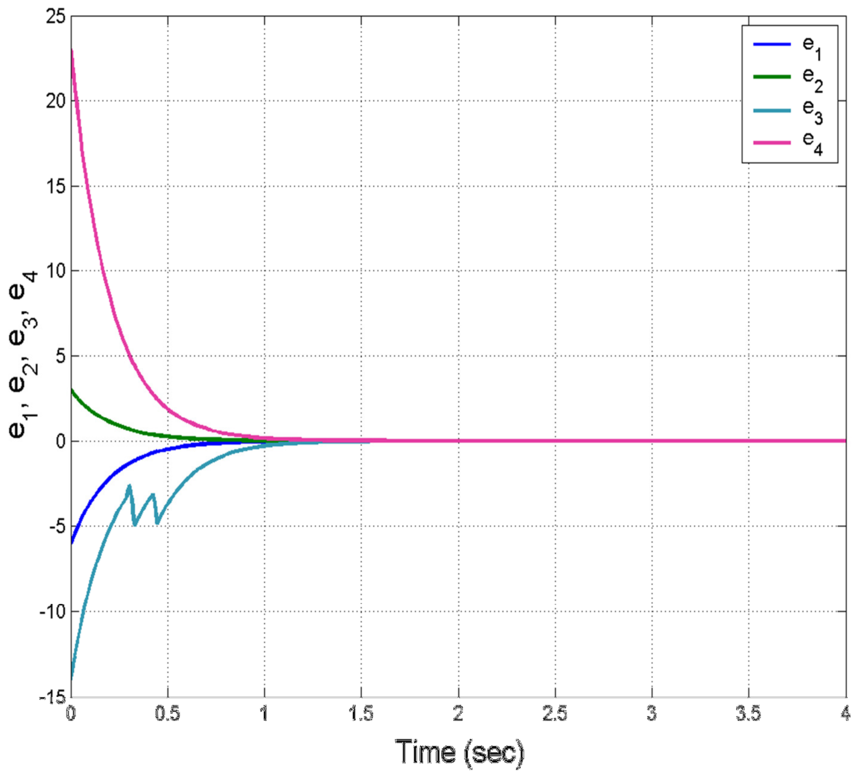

Figure 8. Time-History of the Hybrid Synchronization Errors e_1, e_2, e_3, e_4

7. CONCLUSIONS

This paper derived new results for the active controller design for the hybrid synchronization of hyperchaotic Zheng systems (2010) and hyperchaotic Yu systems (2012). Using Lyapunov control theory, active control laws were derived for globally hybrid synchronizing the states of identical hyperchaotic Zheng systems, identical hyperchaotic Yu systems and non-identical hyperchaotic Zheng and Yu systems. MATLAB simulations were shown for the hybrid synchronization results derived in this paper for hyperchaotic Zheng and Yu systems.

REFERENCES

[1] Rössler, O.E. (1979) "An equation for hyperchaos," *Physics Letters A*, Vol. 71, pp 155-157.

[2] Huang, Y. & Yang, X.S. (2006) "Hyperchaos and bifurcation in a new class of four-dimensional Hopfield neural networks," *Neurocomputing*, Vol. 69, pp 13-15.

[3] Machado, L.G., Savi, M.A. & Pacheco, P.M.C.L. (2003) "Nonlinear dynamics and chaos in coupled shape memory oscillators," *International Journal of Solids and Structures*, Vol. 40, No. 19, pp. 5139-5156.

[4] Tao, Y. (1999) "Chaotic secure communication systems – history and new results", *Telecommun. Review*, Vol. 9, pp 597-634.

[5] Li, C., Liao, X. & Wong, K.W. (2005) "Lag synchronization of hyperchaos with applications to secure communications," *Chaos, Solitons & Fractals*, Vol. 23, No. 1, pp 183-193.

[6] Prokhorov, M.D. & Ponomarenko, V.I. (2008) "Encryption and decryption of information in chaotic communication systems governed by delay-differential equations," *Chaos, Solitons & Fractals*, Vol. 35, No. 5, pp 871-877.

[7] Yassen, M.T. (2008) "Synchronization hyperchaos of hyperchaotic systems", *Chaos, Solitons and Fractals*, Vol. 37, pp 465-475.

[8] Ott, E., Grebogi, C. & Yorke, J.A. (1990) "Controlling chaos", *Phys. Rev. Lett.*, Vol. 64, pp 1196-1199.

[9] Pecora, L.M. & Carroll, T.L. (1990) "Synchronization in chaotic systems", *Phys. Rev. Lett.*, Vol. 64, pp 821-824.

[10] Bowong, S. & Kakmeni, F.M.M. (2004) "Synchronization of uncertain chaotic systems via backstepping approach," *Chaos, Solitons & Fractals*, Vol. 21, No. 4, pp 999-1011.

[11] Suresh, R, & Sundarapandian, V. (2012) "Global chaos synchronization of WINDMI and Coullet chaotic systems by backstepping control", *Far East J. Math. Sciences*, Vol. 67, No. 2, pp 265-287.

[12] Suresh, R. & Sundarapandian, V. (2012) "Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback", *Arch. Control Sciences*, Vol. 22, No. 3, pp 255-278.

[13] Senejohnny, D.M. & Delavari, H. (2012) "Active sliding observer scheme based fractional chaos synchronization," *Comm. Nonlinear Sci. Numerical Simulation*, Vol. 17, No. 11, pp 4373-4383.

[14] Sundarapandian, V. (2012) "Anti-synchronization of hyperchaotic Xu systems via sliding mode control", International Journal of Embedded Systems, Vol. 2, No. 2, pp 51-61.

[15] Sundarapandian, V. (2013) "Anti-synchronizing sliding controller design for identical Pan systems," *International Journal of Computational Science and Information Technology*, Vol. 1, No. 1, pp 1-9.

[16] Huang, L. Feng, R. & Wang, M. (2004) "Synchronization of chaotic systems via nonlinear control," *Physics Letters A*, Vol. 320, No. 4, pp 271-275.

[17] Lei, Y., Xu, W. & Zheng, H. (2005) "Synchronization of two chaotic nonlinear gyros using active control," *Physics Letters A*, Vol. 343, pp 153-158.

[18] Sarasu, P. & Sundarapandian, V. (2011) "Active controller design for generalized projective synchronization of four-scroll chaotic systems", *International Journal of System Signal Control and Engineering Application*, Vol. 4, No. 2, pp 26-33.

[19] Sundarapandian, V. (2012) "Adaptive control and synchronization of a generalized Lotka-Volterra system," Vol. 1, No. 1, pp 1-12.

[20] Sundarapandian, V. (2013) "Adaptive controller and synchronizer design for hyperchaotic Zhou system with unknown parameters," Vol. 1, No. 1, pp 18-32.

[21] Zhao, J. & Lü, J. (2006) "Using sampled-data feedback control and linear feedback synchronization in a new hyperchaotic system," *Chaos, Solitons & Fractals*, Vol. 35, pp. 376-382.

[22] Ma, H., Deshmukh, V., Butcher, E. & Averina, V. (2005) "Delayed state feedback and chaos control for time-periodic systems via a symbolic approach", *Communications in Nonlinear Science and Numerical Simulation*, Vol. 10, No. 5, pp 479-497.

[23] Zheng, S., Dong, G. & Bi, Q. (2010) "A new hyperchaotic system and its synchronization," *Applied Mathematics and Computation*, Vol. 215, pp 3192-3200.

[24] Yu, F., Wang, C.H., Hu, Y. & Yin, J.W. (2012) "Antisynchronization of a novel hyperchaotic system with parameter mismatch and external disturbances", *Pramana*, Vol. 79, No. 1, pp 81-93.

[25] Hahn, W. (1967) *The Stability of Motion*, Springer, Berlin.

Author

Dr. V. Sundarapandian earned his D.Sc. in Electrical and Systems Engineering from Washington University, St. Louis, USA in May 1996. He is Professor and Dean of the R & D Centre at Vel Tech Dr. RR & Dr. SR Technical University, Chennai, Tamil Nadu, India. So far, he has published over 300 research works in refereed international journals. He has also published over 200 research papers in National and International Conferences. He has delivered Key Note Addresses at many International Conferences with IEEE and Springer Proceedings. He is an India Chair of AIRCC. He is the Editor-in-Chief of the AIRCC Control Journals – *International Journal of Instrumentation and Control Systems*, *International Journal of Control Theory and Computer Modelling*, *International Journal of Information Technology, Control and Automation*, *International Journal of Chaos, Computing, Modelling and Simulation* & *International Journal of Information Technology, Modeling and Computing*. His research interests are Control Systems, Chaos Theory, Soft Computing, Operations Research, Mathematical Modelling and Scientific Computing. He has published four text-books and conducted many workshops on Scientific Computing, MATLAB and SCILAB.