A SIMPLE METHOD TO CALCULATE THE ERROR PROBABILITY OF MULTIUSER MULTI-ANTENNA RECEIVERS OVER FREQUENCY RAYLEIGH CHANNEL

Rim Haddad[1], Ridha Bouallègue[2]

Laboratory Research in Telecommuniccation 6'Tel in High School of Communication of Tunis, Route de Raoued Km 3.5, 2083 Ariana, Tunisia

[1] rim.haddad@isima.rnu.tn

[2] ridha.bouallegue@supcom.rnu.tn

ABSTRACT

Smart or adaptive antennas promise to provide significant increases in system capacity and performance in wireless communication systems. Multiuser detectors have the capacity of eliminating MAI, the near-far problem and providing a significant capacity increase. In order to gain from the enhancements of both: multiuser detection and adaptive antenna. We propose in this paper to combine both schemes for an asynchronous systems.Our analysis is based on modeling the angular gain of the spatial filter (array beam pattern) by a piece-line function that approximates the passband (or in-beam) and the stopband (or out-beam) with an equivalent attenuation. The proposed model conforms the benefits of adaptive antennas in reducing the overall interference level (intercell/intracell) and to find an accurate approximation of the error probability.

KEYWORDS

Beamforming, Direction of Arrival (DoA), BER, Rayleigh fading, MUI

1. INTRODUCTION

ONE of the primary limitations on the performance of cellular communication systems is Multiple Access Interference (MUI). Hence, current research activities are focused on reducing this interference. One approach that has shown real promise for substantial capacity enhancement is the use of spatial processing with adaptive antenna arrays. Antenna arrays can be thought of as spatial filters in the sense that they can be used to form a beam toward the desired user while spatially rejecting the interferers outside the beam.

In a typical mobile environment, signals from users arrive at different angles to the base station and hence antenna arrays can be used to an advantage. Each multipath of a user may arrive at a different angle, and this angle spread can be exploited using an antenna array[1,2].

In principle, a multi-user receiver allows constructive combination of multi-path signals received by an array of antennas while minimizing the MAI's contribution. Besides, providing the average error probability for K users with DOA's (Direction of Arrival) uniformly distributed within a symmetric support around the array broadside has been derived for chip and phase asynchronous DS-CDMA system [3,4].

The Bit Error Rate (BER) is considered to be one of the most important performance measures for communication systems and hence it has been extensively studied. The exact analytical evaluation of the probability of error in DS-CDMA, is still an open subject.

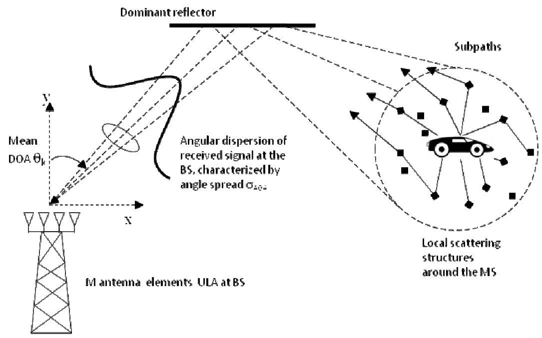


Figure1. Illustration of wireless propagation environment

Gaussian approximations of the Multiuser Interference (MUI) are used to reduce the problem, and to be tractable namely when the average performance is of interest. However, the accuracy of the Gaussian approximation technique depends on the specific configuration of the system. When analyzing the BER performance of DS-CDMA systems, the interference sources, namely the Multiple Access Interference (MAI) are commonly assumed to be Gaussian distributed [2,5].

Hence in this paper, we will derive an accurate BER formula for Nakagamim–fading channel in the context of asynchronous transmission and we propose to adapt the Gaussian approximations to antenna array systems by properly accounting for the noise and the MUI after beamforming. This is carried out by considering an approximation of the angular gain in adaptive antenna array systems. To the best of the author's knowledge, the term adaptive antenna array implies the beamforming of the receiving system is optimized on the basis of the knowledge of the direction of arrivals (DoAs) for all the K users.

In this contribution, we designed differently spatial filter weighs to cope with MUI, the receiver performances are strictly related to the efficiency of MUI reduction. The average performance for users randomly distributed within an angular support depends on the probabilities of each of these DoA alignments. The ratio between the array beamwidth and the angular sector of the impinging DoAs define the efficiency of the array system.

In our paper, we propose a novel approach to evaluate the average probability of error by considering an approximation of the spatial filter. The angular gain function is approximated by a fixed beamwidthθ_{BW}, for the passband and by an attenuation α_0. For the user of interest, all the remaining $k-1$ interferers are partitioned into in-beam / out-beam MUI. The analytical formulas ofthe average error probability is counted differently for the in-beam/out-beam interferers and we validate out research for single antenna systems.

Although involving several approximations, the numerical validation for multi-user receiver with adaptive antennas shows that the model proposed here provides accurate results namely when the number of the in-beam interferers is large (large k and θ_{BW}) and the Gaussian approximation holds true. For this reason, we believe that this simplified approach could be useful to reduce costs in computer simulations when evaluating the average system performance (in term of error probability) in a multi-user system with adaptive antenna array.

We organize the rest of the paper as follows. In section II we introduceour system and channel model, followed by the adaptive antenna description in section III. The error probability with adaptive antenna and numerical results are provided in section IV and section V respectively. We conclude in section VI.

2. System and Channel Model

2.1. The Signal Model

As a preliminary step, let us introduce the scenario wherein the adaptive antenna operates. We consider a DS-CDMA wireless network with K subscribers. The base station is equipped with a uniform linear array (ULA) of M equi-spaced identical elements (Figure 2).

The array receives the signals from the K subscribers located in the far field zone of the array. We assume that all the signals are uncorrelated and each user transmits a binary phase-shift keying (BPSK) symbols.The base band equivalent model is considered for asynchronous modulation waveforms $S_1(t), S_2(t), \dots S_K(t)$.

The transmitted signal of the K^{th} user is:

$$x_k(t) = \sum_i b_k^i S_k(t - iT) \tag{1}$$

$b_k^i \in \{-1, +1\}$is the i^{th} transmitted BPSK symbol and T is the symbol interval. The user's signal $x_k(t)$propagates through a multipath channel, θ_k is the DoA of the k^{th} user.

The impulse response can be written as:

$$\boldsymbol{h}_k(t) = \sum_{m=1}^{L} \alpha_{k,m}\delta\left(t - \tau_{k,m}\right) \tag{2}$$

Wherein $\alpha_{k,m}$and $\tau_{k,m}$ are the complex gain and delay of the m^{th} path. We assume that all the users have the same number of paths L , the delay $\tau_{k,m} \in [0, T[$ have increasing values: $0 \le \tau_{k,1} \le \tau_{k,2} \le \square \le \tau_{k,L} < T$, for $\forall k$.

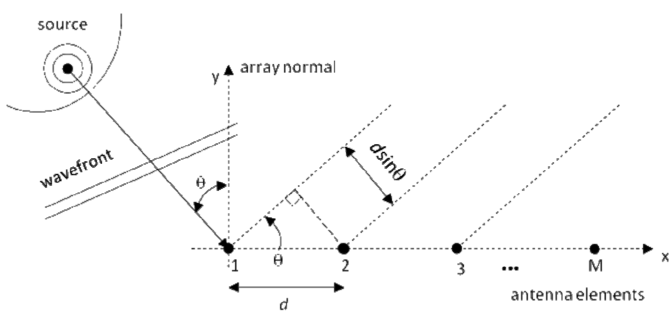


Figure 2. Uniform linear array geometry

In Figure 2,the spatial response of the array due to an incident plane wave from θ_k direction is modeled by the array steering vector $a(\theta_k)$ [6,7].Wherein $a(\theta_k)$ is the $M \times 1$ vector that describes the array response to the DoAθ_k, and the n^{th} element for a linear of half-wavelength spaced antennas is:

$$[a(\theta_k)]_n = exp(-j(n-1)\pi \sin\theta_k) \tag{3}$$

At the receivers, the $M \times 1$ vector of that received signal for the k^{th} user.

$$r_k(t) = a(\theta_k)\boldsymbol{h}_k(t) * x_k(t) \tag{4}$$

$$r_k(t) = a(\theta_k)\sum_i b_k^i \sum_{m=1}^{L} \alpha_{k,m} S_k(t - iT - \tau_{k,m}) \quad (5)$$

The received signal of the K users' signal can be written as:

$$r(t) = \sum_{k=1}^{K} r_k(t) + \sigma n(t) \quad (6)$$

The noise $n(t)$ is assumed to be a zero-mean temporally and spatially uncorrelated Gaussian process, with $E[n.n^*] = I \cdot \delta(\tau)$, σ^2 is the power of the AWGN. We assume that the spatial correlation of noise arising from intercell interference is not considered.

After the beamformingwith the $M \times 1$ spatial filter w_l for the l^{th} user, the output of the l^{th} filter matched to $S_l(t)$ is:

$$y_{n/l}[j] = \int S_l^*(t - \tau_{l,n} - jT) r(t)\, dt \quad (7)$$

$$y_{\frac{n}{l}}[j] = \sum_{k=1}^{K}\sum_i b_k^i G\left(\theta_{\frac{k}{l}}\right)\sum_{m=1}^{L} \alpha_{k,m}\rho_{k,l}\left((i-j)T + \tau_{l,n} - \tau_{k,m}\right) + \sigma n'_{\frac{n}{l}}[j] \quad (8)$$

$G\left(\theta_{\frac{k}{l}}\right) = w_l^H . a(\theta_k)$, where w_l^H is the weight vector and H denotes Hermitian transpose: is the spatial gain of the beamformer designed for the angle θ_l .And $\rho_{k,l}(\tau) = \int S_k(t+\tau) S_l^*(t) dt$ is the cross correlation function between signatures.

The matched filter output contains the self interferenceISI and the MUI.The self interference ISI is written by this equation:

$$y_{\frac{n}{l}}^{ISI}(j) = G\left(\theta_{\frac{l}{l}}\right)\sum_i b_l^i \sum_{m=1}^{L} \alpha_{l,m}\rho_{l,l}\left((i-j)T + \tau_{l,n} - \tau_{l,m}\right) \quad (9)$$

The MUI is:

$$y_{\frac{n}{l}}^{MUI}(j) = \sum_{k=1,k=l}^{K}\sum_i b_k^i G\left(\theta_{\frac{k}{l}}\right)\sum_{m=1}^{L} \alpha_{k,m}\rho_{k,l}\left((i-j)T + \tau_{l,n} - \tau_{k,m}\right) \quad (10)$$

The noise power after the beamformingis $\sigma^2/_M$. To make the analytic evaluation of error probability computation feasible, we have to assume that the waveforms are randomly generated on each BPSK symbol with outcome uniform on $\{-1, +1\}$.

2.2. MUD Receivers with Adaptive Antennas:

The use of adaptive antenna array in MUD receivers is expected to be effective mainly in reducing intercell interference. However, to evaluate the advantage of the array processing in reduction of intercell interference a simplified model can be viewed as a synchronous model with an increased number of fictitious users. Therefore, the synchronous model for one path, $\tau_{1,1} = \tau_{2,1} = \cdots \tau_{K,1} = 0$ for $\forall k$, is considered here as a useful example to gain insights on the array processing gain in MUD [10].

The received signal after spatial filtering and despreading is:

$$y_l[j] = \sum_{k=1}^{K} \sum_{j} b_k^j \; \alpha_{k,1} G\left(\theta_{k/l}\right) \rho_{k,l} + \sigma n'_l[j] \quad ; \text{for } l = 1,2,\dots,K \tag{12}$$

Where $\rho_{k,l} = \rho_{k,l}(\tau_{k,1} = 0)$.

2.3. Channel Model:

The delays $\tau_{k,m}$ and phases $\alpha_{k,m}$ are i.i.d random variables uniformly distributed.The used model for frequency selective multipath channel for mobile communication is the Nakagamimodel, the amplitudes $|\alpha_{k,m}|$ are independent random variables with Nakagami probability density function. $F(|\alpha_{k,m}|, \xi, \sigma_{k,m}^2)$ where:

$$F(|\alpha_1(k,m)|, "(" , \sigma_1(k,m)^{\uparrow}2) = (2(^{\uparrow}(|\alpha_1(k,m)|^{\uparrow}(2" (" - 1))/(\sigma_1(k,m)^{\uparrow}2(\Gamma(()) exp(-("(" |\alpha_1(k,m)|^{\uparrow}2)/ \tag{13}$$

$\Gamma(\cdot)$is the gamma function.The fading parameter $(\text{"}(\text{"} \geq 1/2)$ spans different distributions: The Rayleigh distribution for $(= 1$ while in the limit $\xi \to \infty$ the fading channel converges to a no-fading channel.

To simplify, we assume that the power delay profile of the path strengths is the same for all the users:

$$\sigma_{k,m}^2 = \sigma_{k,0}^2 exp(-\delta(m-1)) \tag{14}$$

The parameter $\delta, \delta \geq 0$ is the decay rate, the total average fading power.

$$\sigma_k^2(\delta) = \sum_{m=1}^{L} \sigma_{k,m}^2 = \sigma_{k,0}^2 q(L,\delta) \tag{15}$$

It depends on the decay rate and the number of paths: $q(L,\delta) = L[1 - exp(-\delta L)] / [1 - exp(-\delta)]$. For $\delta = 0$ the power delay profile is uniform and $q(L,\delta) = L$.

2.4. Problem Statement:

We assume that $l = 1$ be the user of interest, the error probability of DS-CDMA system $BER = P\left(E/_{\theta}, A\right)$depends on the DoAs and fading amplitudes: $\theta = [\theta_1, \theta_2, \cdots, \theta_k, \cdots, \theta_K]^T$ and the set of instantaneous fading amplitudes $A = [\alpha_1, \alpha_2, \cdots, \alpha_k, \cdots, \alpha_K]$.

Where $\alpha_k = [\alpha_{k,1}, \alpha_{k,2}, \dots \alpha_{k,m}, \dots, \alpha_{k,L}]^T$ is the fading set for the k^{th}user. The spatial gain $G\left(\frac{\theta_k}{1}\right)$ has the effect to modify the faded amplitudes as $\frac{\alpha_k}{1} = G\left(\frac{\theta_k}{1}\right)\alpha_k$.

The Bit Error Rate $P\left(E/_{\theta}, A\right)$can be evaluated by any of the known relationship provided withnoise, instantaneous fading signal and interference are appropriately taken into account.

However, the evaluation of the performance (with respect to fading) for any set of DoAs:

$$P\left(E/_{\theta}\right) = E_A\left[P\left(E/_{\theta, A}\right)\right] \tag{16}$$

Can be solved by modifying the fading power for each user/interferers as $\sigma^2_{\frac{k}{1}}(\delta)$, BER (16) can be evaluated as for single-antenna receivers [11].The problem we propose to solve is the evaluation of the average BER for a known distribution of DOAs and instantaneous fading:

$$P\left(E/_{A}\right) = E_{\theta}\left[P\left(E/_{\theta, A}\right)\right] \tag{17}$$

Or the average BER:

$$P(E) = E_{\theta}\left[E_{A}\left[P\left(E/_{\theta, A}\right)\right]\right] \tag{18}$$

The main difficulty in evaluating the error probability (17) or (18) lies in the fact that: 1) different beamforming strategies can be employed; and 2) spatial gain $G\left(\theta_{\frac{k}{1}}\right)$ is a non-linear function of the DoAs. In our paper, we assume that the approximation of the array gain $G\left(\theta_{\frac{k}{1}}\right)$ is for conventional beamforming ($w_1 = \frac{a(\theta_1)}{M}$).

The MUI can be reduced to two dominant terms: the in-beam interferers are not attenuated by the spatial filter and those that are attenuated are out-beam interferers. This partitioning is used to approximate the average BER as it largely simplifies the evaluation of the contribution of the MUI.In this paper, we restrict our analysis to the case of users with i.i.dDoA'sθ_k with pdf $f_{\theta}(\theta_k)$ supported within $\left[-\frac{\Delta\theta}{2}, +\frac{\Delta\theta}{2}\right]$.

The average performance is evaluated with no-fading channel and over Nakagami m-fading channel in section IV, for random spreading sequence of length N, rectangular chip-shaping, equi-power users.The performance evaluation is considered in closed form for the following settings:

1) $P\left(E/_{A}\right)$for chip and phase asynchronous CDMA in no-fading channel ($L = 1$).
2) $P(E)$for$L = 1$ (flat fading) and $L > 1$ (frequency-selective fading) with the assumption that all the users have the same paths strength $\sigma^2_{k,0} = \sigma^2_A$.

3. ADAPTIVE ANTENNA DESCRIPTION:

3.1.Adaptive Antenna Arrays Criteria:

An antenna array is defined as a group of spatially distributed antennas. The output of the antenna array is obtained by properly combining each antenna output. By this operation, it is possible to extract the desired signal from all received signals, even if the same frequency band is occupied by all signals. An antenna array can reduce the interference according to the directions of arrival DoA. Even if the delay time is large, the system complexity does not increase because the antenna array can reduce the interference by using the antenna directivity [12,13].

Therefore, the adaptive antenna array implies that the beamforming of the receiving system is designed to optimize the received power on the basis of the knowledge of the DoA for all the K users.

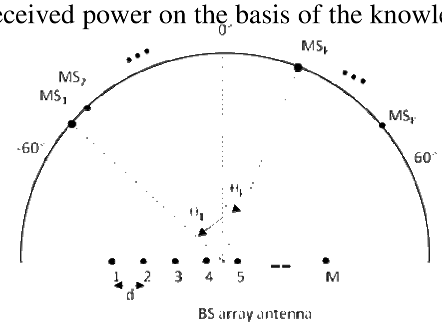


Figure 3.Smart antenna BS serving a single 120° angular sector

It is assumed that the BS employs a Uniform Linear Array (ULA) of M identical omni-directional antenna elements, with inter-element spacing of d=λ/2, as shown in Figure 3.

We will consider the optimal weight beamformer w_1 for the DoA of interest θ_1 must be capable to pass undistorted the signals with DoA θ_1 and to attenuate all the other DoA's different from θ_1. Obviously the receiver performances are strictly related on the efficiency of MAI reduction, but for a number of users $K > M$ there are not enough degrees of freedom to cancel all the interferers. The optimal weight for the user of interest can be reduced to [14] :

$$w_1 = \frac{Q^{-1}a(\theta_1)}{a(\theta_1)^H Q^{-1} a(\theta_1)} \tag{19}$$

The $M \times M$ matrix Q depends on the criterium adopted. Conventional beamforming $w_1 = \frac{a(\theta_1)}{M}$ can be obtained for $Q = I$ .The array power must maintain a unit response in the desired direction, for $= Q_S$, where Q_S is the spatial covariance matrix.

$$Q_S = E_A[r(t).r(t)^H] = \sum_{k=1}^{K} \sigma_k^2 a(\theta_k) a(\theta_k)^H + \sigma^2 I \tag{20}$$

The BER performance depends on the in-beam interferers (section IV). It is interesting to evaluate the capability of the beamforming to cancel an interferer having a DoA (say, θ_2) close to the desired user θ_1.

To simplify our work, we consider the case of a large number of users with a uniform distribution of DoAs and all with the same power $\sigma_k^2(\delta) = \sigma_A^2$ (forL$= 1$) the covariance matrix can be simplified as:

$$Q_S \approx \sigma_A^2[a(\theta_1)a(\theta_1)^H + a(\theta_2)a(\theta_2)^H + K_{eq} I] \tag{21}$$

Where $K_{eq} = \left(K - 2 - \sigma^2/\sigma_A^2\right)$

In the next section, we consider the equivalent spatial filter for conventional beamforming only.

3.2. Array Gain Approximation:

For the user of interest (say θ_1) conventional beamforming weights are considered $w_1 = a(\theta_1)/M$, the angular gain function for conventional beamforming is $G\left(\theta/\theta_1\right) = a(\theta_1)^H a(\theta)/M$.

To simplify the computations in the following, the gain $\left|G\left(\theta/\theta_1\right)\right|^2$ can be approximated by a piece-line function $\left|G_{eq}\left(\theta/\theta_1\right)\right|^2$ that models the pass-band or in-beam with support $f_{DOA}(\theta_1) = [\theta_1 - \theta_{BW}(\theta_1), \theta_1 + \theta_{BW}(\theta_1)]$ with a linear gain and the out-beam (with support $\bar{f}_{DOA}(\theta_1)$ with an equivalent attenuation α_0) [11].The gain can be approximated by:

$$\left|G_{eq}(\theta/\theta_1)\right|^2 = \begin{cases} 1 - \frac{1}{2}\frac{|\theta-\theta_1|}{\theta_{BW}(\theta_1)} & for \quad \theta \in f_{DOA}(\theta_1) \\ \alpha_0 & for \quad \theta \in \overline{f}_{DOA}(\theta_1) \end{cases} \quad (22)$$

The beamwidth $\theta_{BW}(\theta_1)$ depends on the number of antennas M and θ_1. The support $\theta = f_{DOA}(\theta_1) \cup \bar{f}_{DOA}(\theta_1) = \left[{-\Delta\theta}/{2}, {\Delta\theta}/{2}\right]$ covers all the admissible DoAs (in a mobile system with three-cell sectorization the angles range in $\pm 60^0$).

For small deviations from the broadside (for $\theta_1 1 \approx 0$ deg) , the beamwidth θ_{BW} optimized for $\theta_1 = 0$ can be transformed into the beamwidth for any value θ_1 by [15]:

$$\theta_{BW}(\theta_1) = \frac{\theta_{BW}}{\cos\theta_1} \quad (23)$$

By using this model, the in-beam interferers can be easily evaluated for any DoA of interest θ_1 as in those in the support $f_{DOA}(\theta_1)$.The approximation parameters are expected to overestimate the average BER when employing minimum variance beamforming with a small number of users K.

4. ERROR PROBABILITY WITH ADAPTIVE ANTENNA:

The effect of spatial filter is to enhance the differences in the power of the interfering users.The interferers are partitioned into two interference driven spatial equivalence classes: the in-beam and the out-beam interferers depending on whether the users have the in-beam and out-beam DoAs with respect to the user of interest [16].

For each user of interest characterized by the DoA θ_1, the remaining $K-1$ users are partitioned into the two disjointed subsets:

$$\{2,3\cdots,K\} = B(\theta_1) \cup \bar{B}(\theta_1) \quad (24)$$

such that $\theta_j \in f_{DOA}(\theta_1)$ if $j \in B(\theta_1)$ within each spatial equivalence class the users have (approximately) the same power.

The in-beam and the out-beam users contribute to the level of interference according to the cardinality of each set, $|B(\theta_1)|$ and $|\bar{B}(\theta_1)|$ respectively [2,4]. The $|B(\theta_1)|$ in-beam users contribute to the overall level of interference at the decision variable, the remaining $|\bar{B}(\theta_1)| = K - 1 - |B(\theta_1)|$ asynchronous users can be assimilated to a Gaussian noise and thus contribute to modifying the decision variable.In this case, the AWGN can be increased according to the “Standard Gaussian approximation”.

According to the spatial filter approximation (18) all the $|B(\theta_1)|$ interferers experience an attenuation that is within $\left[{1}/{2}, 1\right]$. The support $f_{DOA}(\theta_1)$ is small, the pdf of the DoAs of the in-beam interferers conditioned to the support $f_{DOA}(\theta_1)$ is almost uniform.

The remaining $|\bar{B}(\theta_1)| = K - 1 - |B(\theta_1)|$ users are attenuated by α_0.The power of the overall instantaneous fading interference is σ_I^2 and it depends on the combination of the (in-beam/out-beam) MUI.

After the beamforming the computation of the BER can be derived by taking into account the instantaneous fading amplitudes α_1, the overall noise and instantaneous interference. The conditional BER is:

$$P\left(E/_{\theta_1}, A\right) = \sum_{|B(\theta_1)|=0}^{K-1} P\left[E/_{A}, |B(\theta_1)|\right] p(|B(\theta_1)|) \tag{25}$$

$p(|B(\theta_1)|)$ denotes the probability of the cardinality for the subset $B(\theta_1)$ and $P\left(E/_{A}, |B(\theta_1)|\right)$ is the BER for a specific receiver independent on the DoAs.To simplify the notation, the cardinality of the in-beam interference is $K_I = |B(\theta_1)|$.

4.1. No-fading channels:

We consider a system with $L = 1$ and $A = [\alpha_1, \alpha_2, \dots, \alpha_K]$, the error probability $P_e\left[A^2, \sigma^2/_{M}, K_I\right] = P\left[E/_{A}, |B(\theta_1)|\right]$ depends only on the number of in-beam interferers K_I and not on their DoAs.

The average error probability reduces:

$$P(E) = \sum_{K_I=0}^{K-1} P_e\left[A^2, \frac{\sigma^2}{M}, K_I\right] \int_{-\Delta\theta/_2}^{\Delta\theta/_2} p(K_I) f_\theta(\theta_1) d\theta_1 \tag{26}$$

$p(K_I)$ is the probability of having K_I in-beam interferers. The probability of an In-beam interferer:

$$p(\theta \in \theta(\theta_1)) = \int_{\theta_1-\theta_{BW}(\theta_1)}^{\theta_1+\theta_{BW}(\theta_1)} f_\theta(\xi) d\xi \tag{27}$$

Depends on the beamwidth $\theta_{BW}(\theta_1) = \theta_{BW}/_{\cos\theta_1}$.For DOAs uniformly distributed within the support $\left[-\Delta\theta/_2, +\Delta\theta/_2\right]$, the probability (27) depends on the beamwidth $\theta_{BW}(\theta_1)$ compared to the overall support θ .

$$p(\theta \in \theta_1) = \frac{2\theta_{BW}(\theta_1)}{\Delta\theta} = \frac{\eta}{\cos\theta_1} \tag{28}$$

Where $\eta = \frac{2\theta_{BW}}{\Delta\theta}$ depends on the beamforming criterion exploited.

The average BER (26) becomes:

$$P(E) = \sum_{K_I=0}^{K-1} \eta^{K_I} \binom{K-1}{K_I} \chi(\eta, K, K_I) P_e\left[A, \frac{\sigma^2}{M}, K_I\right] \tag{29}$$

Where $\chi(\eta, K, K_I) \approx (1-\eta)^{K-K_I-1}$ for matched filter receiver with $L = 1$ branch the BER depends on the interference [11]:

$$P_e\left[A, \frac{\sigma^2}{M}, K_I\right] = Q\left[\left(\frac{\sigma^2}{MA^2} + \frac{\sigma_I^2(K_I)}{A^2}\right)^{-1/_2}\right] \tag{30}$$

$Q[\cdot]$ is the Gaussian Q-function.The average level of interference (for chip and phase asynchronous)[9]:

$$\sigma_I^2(K_I) = \frac{3}{4} A^2 \frac{K_I}{3N} + \alpha_0 A^2 \frac{K - K_I - 1}{3N} \tag{31}$$

Counted for the K_I in-beam and the $(K - K_I - 1)$ out-beam users. For small values of η, the number of in-beam interferers is small. The more accurate approximation in [17] could be used for receivers with adaptive antennas.

For MMSE-MUD receivers with adaptive antenna arrays, the BER depends only on the in-beam users as the attenuated out-beam users are approximately decoupled. The evaluation of BER for the user of interest [4]:

$$P_e\left(A^2, \frac{\sigma^2}{M}, K_I\right) = Q\left[\left(\frac{MA^2}{\sigma^2} - \frac{\frac{MA^2}{\sigma^2}\rho^2\frac{3}{8}K_I}{1 + \frac{\sigma^2}{MA^2} + \rho\left(\frac{3}{8}K_I - 1\right)}\right)^{1/2}\right] \tag{32}$$

The Gaussian approximation is based on the asymptotic analysis for $K_I \to \infty$ is used to derive (32), it is accurate enough to yield meaningful conclusions in the evaluation of the benefits of the MMSE-MUD in a receiving system employing an adaptive array system, and it decouples the effects of the spreading signatures in the analysis.

The array gain $1/M$ for the AWGN is included either in the BER for matched-filter receiver (31)and in MMSE-MUD (28). In order to compare the benefits of the spatial diversity for a varying number of antennas, the SNR is measured after the beamformingas $MA^2/2\sigma^2$.

The average BER for any arbitrary DoAs distribution is conceptually dependent on $p(\theta \in f_{DOA}(\theta_1))$ to get $p(K_I)$ for each θ_1 and then averaging with respect to θ_1 (26). The DoA's distribution might become a design parameter.

This occurs when MAI reduction is obtained by dynamically assigning the radio resource according to their DoAs.

4.2. Nakagami m-fading channels:

In this section, we extend the concepts discussed above to the Nakagami m-fading case. The average performance needs to be evaluated from (25) by averaging with the Pdf of fading. The error probability $P\left(E/_A, K_I\right)$ depends on the instantaneous fading SNR $\gamma(\alpha_1, K_I)$ and on the number of in-beam interferers K_I.

Under the Gaussian approximation, the SNR at the decision variable has been evaluated by Eng and Milsten [5] for propagation over Nakagami fading channels.

$$\gamma(\alpha_1, K_I) = \sum_{m=1}^{L} |\alpha_{1,m}|^2 \times \left(\frac{\sigma^2}{2M} + \sigma_A^2 q(L,\delta)\frac{\frac{3}{4}K_I + \alpha_0(K - K_I - 1)}{3N} + \sigma_A^2\frac{q(L,\delta) - 1}{2N}\right)^{-1} \tag{33}$$

Let the number of resolvable multipaths of each user be L , for $L = 1$ (no multipath), the last term in (33) vanishes as $q(L,\delta) = 1$; if $\delta = 0$ it is $q(L,\delta) = L$ and the overall level of MUI and ISI (self interference) increases according to the degree of time diversity: L .

The error probability depends on the desired user θ_1 and the number of in-beam interferers K_I is $P\left(E/_{\theta_1}, K_I\right)$ can be evaluated in term of effective SNR: [5]

$$\bar{\gamma}(K_I) = \frac{q(L,2\delta)}{2\xi q(L,\delta)} \times \left(\frac{\sigma^2}{2\sigma_A^2 M} + q(L,\delta)\frac{\frac{3}{4}K_I + \alpha_0(K-K_I-1)}{3N} + \frac{q(L,\delta)-1}{2N} \right)^{-1} \tag{34}$$

The equation $P\left(E/_{\theta_1}, K_I\right) = P_e\left(\sigma_A^2, \sigma^2/_M, K_I\right)$ is considered only for the case of Rayleigh fading (ξ=1) and flat delay profile $\delta = 0$:

$$P_e\left(\sigma_A^2, \sigma^2/_M, K_I\right) = \frac{1}{2}\left[1 - \mu(K_I)\sum_{l=0}^{L-1}\binom{2l}{l}\left(\frac{1-\mu^2}{4}\right)^l\right] \tag{35}$$

Where: $$\mu(K_I) = \sqrt{\frac{\bar{\gamma}(K_I)}{1+\bar{\gamma}(K_I)}} \tag{36}$$

To evaluate the average BER, we sum over the cardinality of the in-beam set and average with respect to θ_1 like the equation (30).

For uniformly distributed DoAs, it is:

$P(E) = \Sigma_1(K_1I = 0)^{\uparrow}(K-1)$ ▒〖(↑(K₁I) (■(K − 1@K₁I))" (" ("(" , K, K₁I)P(σ↓A↑2, σ↑2 / M, K₁I)〗 (37)

which is dual equation of (29) for fading channels.

5. COMPUTATION RESULTS AND DISCUSSION:

In this section, we carry out the simulatedresults that have been obtained by applying a model of spatial filter that allows todescribe the angular gain function: the in-beam is approximated with a fixed beam-width and the out-beam with an equivalent attenuation.

The BER approximation proposes to adapt the single antenna performance bounds to array antenna systems. This is achieved by manipulating those terms in the error probability formulas for single antenna receivers that account for the noise and MAI.

The approximations used to evaluate the average performance are validated here with numerical results. In the proceeding simulations, the following assumptions are made for all the users:

- The interferers are uniformly distributed in the coverage area comprising an angular sector of 120^0.
- The interferers are partitioned into two spatial equivalence classes: in-beam and out-beam based on whether their Directions of Arrivals lie inside or outside the beam formed toward the desired user.
- Because of the piece-line approximation, the energy of each in-beam interferer is a random variable uniformly distributed within [1/2,1] (half power beamwidth region).
- The multipath channel parameters: number of paths L , DoAs θ_k are independent and uniformly distributed.
- All the users are received with the same average power as in a system with a perfect power control.
- The M omni-directional antennas are arranged in a uniform linear array half wavelength spaced based on the conventional beamforming.
- The SNR is measured after the beamforming so that the $1/_M$ array gain for AWGN is implicitly compensated to focus the attention on the gain arising from spatial diversity.

Figure4 illustrates the average probability versus SNR (solid line) for $L = 1$ branch, $M = 8$ antennas, $K = 8$ and 16 users (N=31) for no-fading channel.

Figure5 shows the average BER using the same parameters as in Figure4 in Rayleigh fading channel$A = [\alpha_{1,1}, \alpha_{2,1}, \ldots, \alpha_{K,1}]$.

Numerical experiments show that receiver with adaptive antennas performance degrades down to single antenna receiver when the spatial filtering is in no way effective in reducing the MUI (for low SNR).

Simulation results are close to the analytical results proposed in this paper by accounting for the effects of the in-beam / out-beam interferers.

Besides, the receivers based on the adaptive arrays demonstrates the efficacy in reducing the overall interference level.

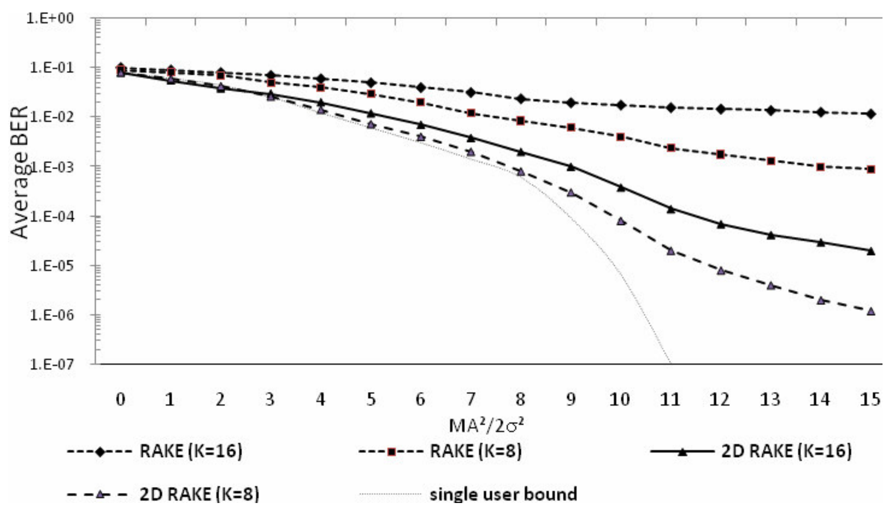


Figure 4. Average BER versus SNR for no fading channel for L=1 path, M=8 antennas, K=8 and 16 users

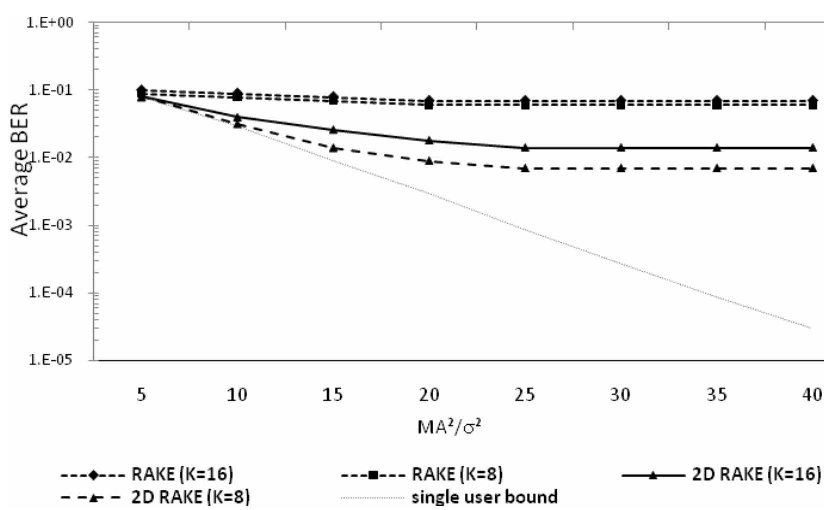


Figure 5. Average BER versus SNR in Rayleigh fading channel for L=1 path, M=8 antennas, K=8 and 16 users

The influence of the interference can be reduced by decreasing the probability of having an in-beam interferer η. This is illustrated in Figure 6, where the BER performance is shown by varying the number of users for no-fading and Rayleigh fading channels.

From Figure 6, it can be noticed that the same average BER can be obtained by doubling the number of antennas M and the number of users K either for no-fading or fading channels.

Therefore, as a rule, the average performance (or the level of the in-beam interference) remains the same as far as the ratio $\frac{M}{K}$ remains constant. This conclusion can be shown even when we neglect the influence of the out-beam interference (by setting $\alpha_0 = 0$ in (29)).

Figure 7 investigates the average BER for propagation over L paths frequency selective Rayleigh fading channel (for L=1,2,4) versus SNR (M=8, K=16, N=31) and versus number of users in Figure 8 (SNR=10dB, N=64) for 2D-RAKE with M=8 (solid lines) and M=16 (dashed lines) antennas, single-antenna RAKE (dotted lines).

The figures Figure 7 and Figure 8, show either for varying SNR ($SNR = {M\sigma_A^2}/{\sigma^2}$) or increasing number of users, that multipath channels (large L) and angular diversity can improve satisfactory performance when exploited jointly.

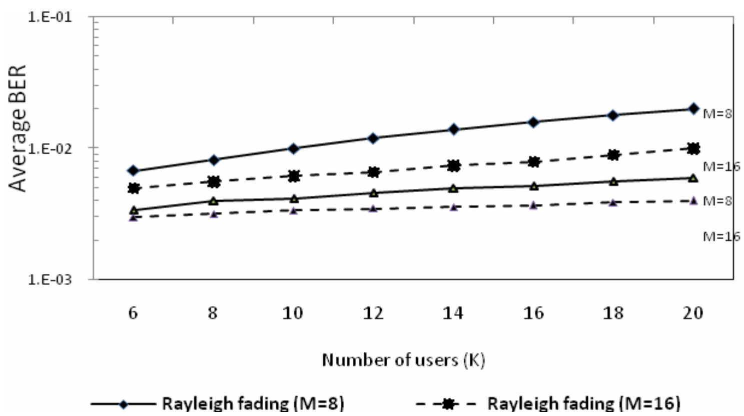


Figure 6. Average BER versus the number of users K for no fadingand Rayleigh fading channels for L=1 path, M=8 and 16 antennas

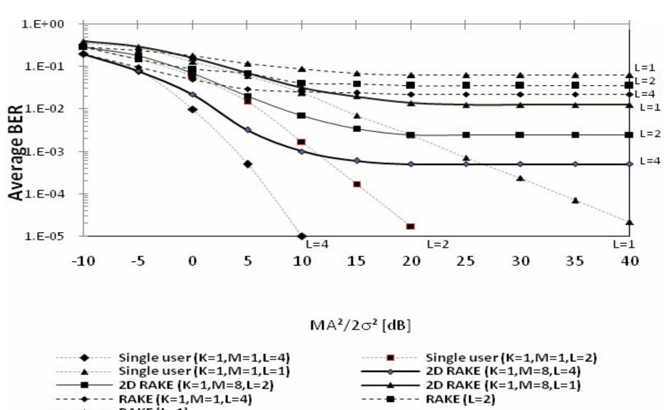


Figure 7. Average BER versus SNR ; M=8 antennas and K=16,single antenna (M=1) and single user/single antenna in multi-path Rayleigh fading channels (L=1, 2, 4)

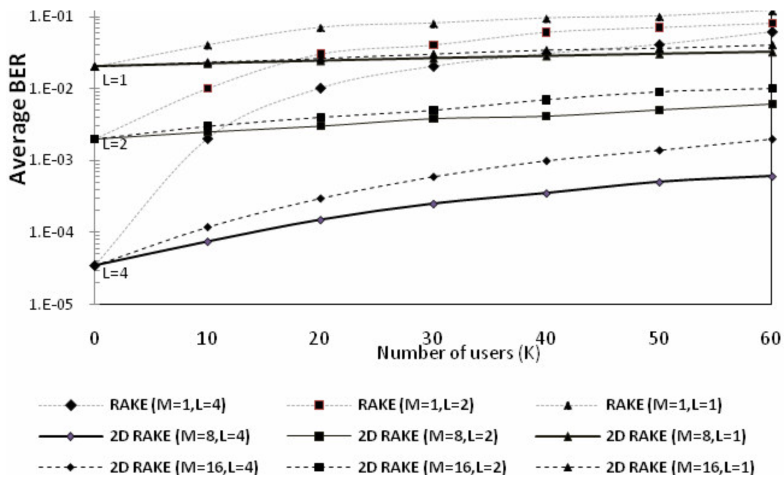


Figure 8. Average BER versus the number of users K with M=8 and 16 antennas

Figure 9 illustrates the average probability of error of MMSE-MUD receivers for synchronous CDMA system with adaptive array and real-valued equi-correlated spreading sequences ($\rho = 0{,}5$), M=8, 16 antennas and no-fading channels. The performance versus SNR for K=16 users shows that there is only a slight additional advantage on the MUI reduction capability when employing an adaptive antenna arrays, even for a high correlation ($\rho = 0{,}5$).

The comparison with the analytic performance ofAdaptive antenna receiver confirms that MUI cancellation of the MMSE-MUD outperforms the receiver for high SNRs.

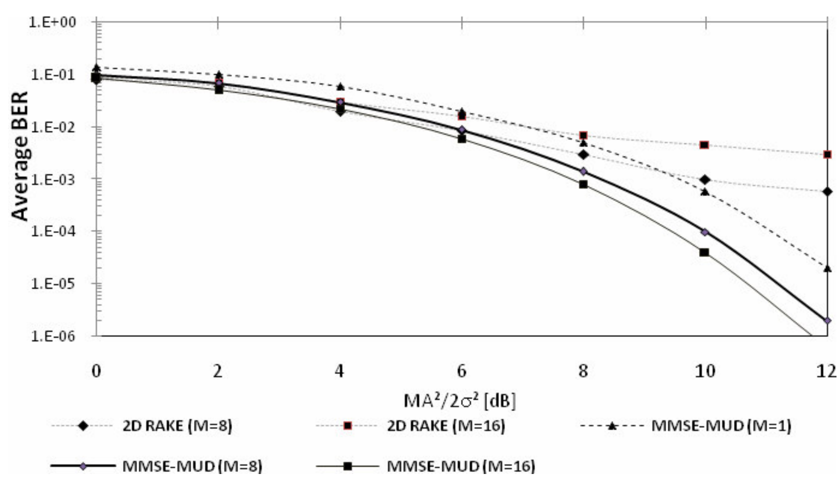


Figure 9. Average BER of MMSE-MUD receivers for synchronous CDMA system with adaptive array versus SNR for K=16 users and ρ=0,5

In Figure 10, the BER is plotted versus the number of users K for $10\ log\left(MA^2/_{2\sigma^2}\right) = 6dB, \rho = 0{,}25\ and\ 0{,}5$. The plot shows that when increasing the correlation $(\rho = 0{,}25\ versus\ \rho = 0{,}5)$, the MUI reduction by the spatial filtering becomes more effective.

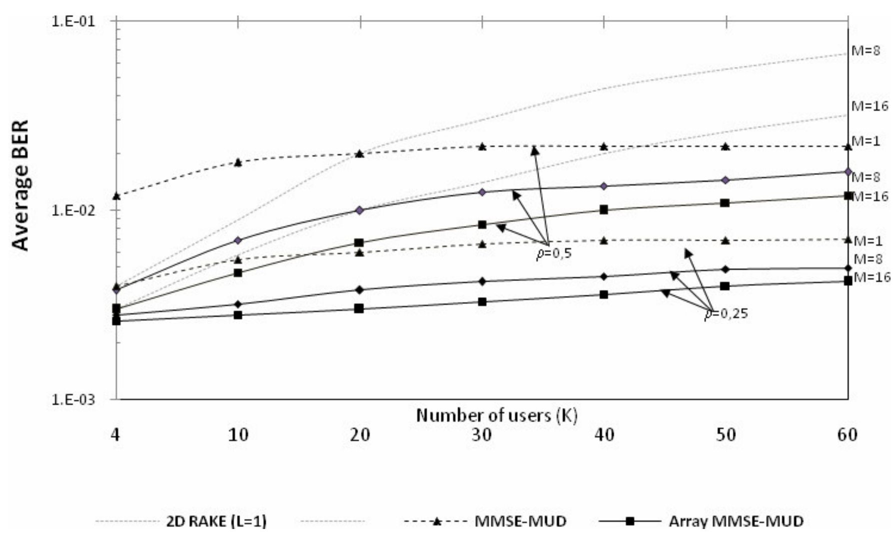


Figure 10. Average BER versus the number of users K for SNR= 6dB and ρ=0,25 and 0,5

6. Conclusions:

This paper has treated the effects of the spatial filter for adaptive uniform linear arrays by a simple model which evaluates the average interference. The simplified approach proposed here handles differently the in-beam and the out-beam interferers by adapting the analytical formulas of the average error probability for single antenna system.

The partition of the users into in-beam and out-beam interferers can largely simplify the analysis since interferers are counted differently in average error probability evaluation.

An important parameter of the Adaptive antenna receiver with regards to quality and capacity of reducing interference is the number of users. That's why; we focused our work in varying the number of users and also to compare the BER performance.

Also, we showed in simulations that we can influence in the number of antennas to evaluate BER. The BER is expected to fall well below the optimum when more number of antennas is used, but with a trade-off of increased cost and complexity. Besides, we noticed that the average performance (or the level of the in-beam interference) remains the same as far as the ratio M/K remains constant.

We have also shown that the multi-antenna receiver rejects interferers both spatially as well as temporally and achieves a BER performance that can come close to the performance of receiver in the single user case (no interferers).

A continuation of the study, which we have already started, is to evaluate the average BER in forward link (base to mobiles) where each user experience the same temporal channel for all the received signals. In this case, the beamforming at the base station is decoupled from the receiver at the mobile terminals. In addition, the beamforming design could be synergic with the MUD receiver at mobiles.

REFERENCES

[1] D. Piazza, N.J. Kirsh, A. Forenza, “Design and Evaluation of a reconfigurable antenna array for MIMO systems”, IEEE Transactions on Antennas and Propagation, vol.56, n°3, pp. 869-881, March 2008.

[2] R. Haddad, R. Bouallègue, “BER Performance in DS-CDMA Systems using a Beamformer”, IEEE International Symposium on Industrial Electronics, vol.1, pp. 585-588, July 2006.

[3] R.Haddad, R.Bouallègue, “BER Performance in Space-Time Processing receiver using Adaptive Antennas over Rayleigh Fading Channels”, Proc. IEEE International Conference on signal Processing and Communication , pp.1483-1486 Nov. 2007.

[4] R.Haddad, R.Bouallègue, “BER Performance of Smart Antenna Systems Operating over Rayleigh fading Channels”, proc. IEEE Wireless Days 2008, pp.1-5, Nov. 2008.

[5] T. Eng and L.B. Milstein, “Coherent DS-CDMA performance in Nakagami multi-path fading”, IEEE Trans. Commun. Vol.43, pp.1134-1143, Feb/March/Apr 1995.

[6] Li Zhang, NingXie, “Non linear optimization for adaptive antenna array receivers with a small data-record size”, Wireless Communications & Mobile Computing, vol.9, pp. 239-249, Feb. 2009.

[7] H. J. Li and T. Y. Liu, “Comparison of beamforming techniques for W-CDMA communication systems”, IEEE Transactions on Vehicular Technology, vol.52, no.4, pp. 752-760, July 2003.

[8] S. Choi, J. Choi, H.J. Im, and B.Choi, “A novel adaptive beamforming algorithm for antenna array CDMA systems with strong interferers”, IEEE Transactions on Vehicular Technology, vol. 51, no. 5, pp. 808-816, Sept. 2002.

[9] X. Wang and H.V.Poor, "Space-Time Multiuser Detection in Multi-path CDMA channel", IEEE Trans. on Comm. vol. 47, pp.185-190, Feb.1999.

[10] B.H. Khalaj, A. Paulraj and T. Kailath, “2D RAKE receivers for CDMA cellular system” Proc. IEEE GLOBECOM conf., pp400-404, Dec. 1994.

[11] U. Spagnolini, “A simplified model for probability of error in DS-CDMA systems with adaptive antenna arrays”, in Proc. IEEE International Conference on Communications (ICC), June 2001, pp.2271-2275.

[12] S. K. Yong and J.S Thomson, “The impact of angle of arrival distribution on the performance analysis of different antenna array architectures”, in Proc. IEE International Conference on Antennas and Propagation (ICAP), vol. 2, no 491, pp. 840-843, Mar. 2003.

[13] S. Durrani and M. E. Bialkowski, “Investigation into the performance of adaptive array in cellular environment”, in Proc. IEEE International Symposium on antennas and Propagation (AP-S), vol. 2, pp. 648-651, June 2002.

[14] S. Durrani and M. E. Bialkowski, “A smart antenna model incorporating an azimuthal dispersion of received signals at the base station of a CDMA system”, IEEE International Multi Topic Conference (INMIC), pp.218-223, Dec. 2003.

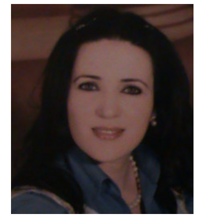
[15] D. H. Johnson and D. E. Dudgeon, Array Signal Processing: Concepts and Techniques. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[16] J, -A. Tsai, R. M. Buehrer, and B. D. Woerner, “BER performance of a uniform circular array versus a uniform linear array in a mobile radio environment”, IEEE Transactions on Wireless Communications, vol. 3, no. 3, pp.695-700, May. 2004.

[17] J. Holtzman, "A simple accurate method to calculate spread-spectrum multiple access error probabilities", IEEE trans. on Comm., vol.40, pp.461-464, Mar. 1992

[18] M.Migliore, D. Pinchero, “Improving Channel Capacity Using Adaptive MIMO Antennas”, IEEE Transactions on Antennas and Propagation, vol.54, n°11, pp. 3481-3489, Nov 2006.

Authors



Rim Haddad (M'05) Received the B. Eng. Degrees in Telecommunication from the High School of Telecommunication of Tunis (SUP'COM). From 1997 to December 2006, she was Engineer in Telecommunication in Tunisie Telecom. Since January 2007 she was an university assistant in the High School of Computing, she has taught courses in telecommunications and security computing. She is currently working toward the Ph.D. degree in Telecommunication systems also at the High School of Telecommunication of Tunis in the Laboratory research of System Telecommunication (6'Tel).

She has worked in several areas including smart antennas, space-time receivers, and angular gain. She's current research interests include smart antenna systems, compact antenna design, and propagation characterization for wireless communications. Ms. Rim won the best paper award of the IEEE-ISIE conference in Montreal Canada.

Ridha Bouallègue (M'98) received the Ph.D degrees in electronic engineering from the National Engineering School of Tunis. In Mars 2003, he received the Hd.R degrees in multiuser detection in wireless communications. From September 1990 he was a graduate Professor in the higher school of communications of Tunis (SUP'COM), he has taught courses in communications and electronics. From 2005 to 2008, he was the Director of the National engineering school of Sousse. In 2006, he was a member of the national committee of science technology. Since 2005, he was the laboratory research in telecommunication Director's at SUP'COM
From 2005 he served as a member of the scientific committee of validation of thesis and Hd.R in the higher engineering school of Tunis. His current research interests include wireless and mobile communications, OFDM, space-time processing for wireless systems, multiuser detection, wireless multimedia communications, and CDMA systems.