

CANCELLATION OF WHITE AND COLOR NOISE WITH ADAPTIVE FILTER USING LMS ALGORITHM

¹Solaiman Ahmed, ²Farhana Afroz, ¹Ahmad Tawsif and ¹Asadul Huq

¹Department of Electrical and Electronic Engineering,
University of Dhaka, Bangladesh

²Faculty of Engineering and Information Technology,
University of Technology, Sydney, Australia

ABSTRACT

In this paper, the performances of adaptive noise cancelling system employing Least Mean Square (LMS) algorithm are studied considering both white Gaussian noise (Case 1) and colored noise (Case 2) situations. Performance is analysed with varying number of iterations, Signal to Noise Ratio (SNR) and tap size with considering Mean Square Error (MSE) as the performance measurement criteria. Results show that the noise reduction is better as well as convergence speed is faster for Case 2 as compared with Case 1. It is also observed that MSE decreases with increasing SNR with relatively faster decrease of MSE in Case 2 as compared with Case 1, and on average MSE increases linearly with increasing number of filter coefficients for both type of noise situations. All the experiments have been done using computer simulations implemented on MATLAB platform.

KEYWORDS

Adaptive Noise Canceller, Color Noise, LMS, MSE, Number of Iterations, SNR, Tap Size, White Gaussian Noise

1.INTRODUCTION

Extracting the speech signal of interest from the noise-corrupted signal is an important signal processing operation in voice communication systems. A frequently encountered problem in communication system is the contamination of the useful signals by unwanted signals or noise. In noise cancellation, signal processing operations involve to filtering out the unwanted noise or interference from the signal contaminated by noise so that the desired signal can be recovered. The spectral characteristics of noise is time varying and unknown in many circumstances. In addition, noise power may exceed the power of the useful signal. In such situations, adaptive digital filters can show better performance in cancelling background noise as compared with

conventional non-adaptive filters. In adaptive noise cancelling system, noise cancellation becomes an adaptive process i.e. the system gets adjusted itself according to the changing environment. Adaptive noise cancellation is a process of subduing background noise from the desired signal in an adaptive manner so as improved SNR (Signal to Noise Ratio) can be ensured at the receiving end [1, 2]. The use of adaptive digital filter in noise cancelling system is at the core for achieving this adaptation capability of the system. In adaptive filter, the filter coefficients can be modified intelligently using adaptive algorithms so that the filter can keep track to the instantaneous changes being occurred in its input characteristics [3]. A range of adaptive algorithms has been proposed to achieve optimum system performance in many applications. Some of the proposed adaptive algorithms can be found in [4-10]. In this paper, a comparative study of eliminating white Gaussian noise and colored noise employing LMS algorithm will be reported. Moreover, the convergence curves as well as the effects of number of iterations, SNR and filter taps on the performance of the system will be evaluated considering both type of noise situations.

The rest of the paper is organized as follows. Section 2 provides a brief review of noise in communication system. Adaptive noise cancellation process is explained in Section 3 followed by illustration of LMS algorithm in Section 4. The simulation parameters and results are discussed in Section 5. Finally, this paper is concluded with Section 6.

2.NOISE IN COMMUNICATION SYSTEM

Noise is random in nature. It is unwanted form of energy that enters the communication system and interferes with the information signal. Noise degrades the level of quality of the received signal at the receiver. Noise can be classified as internal noise and external noise. White noise is a Gaussian noise which exists in all frequencies, whereas colored noise exists in some bands of frequencies.

2.1. White Gaussian Noise

The common source of noise which affects communication is usually white noise. The probability density function of white noise is normal distribution known as Gaussian distribution. Its power spectral density is flat and occupies all frequency. In signal processing, a random signal is considered "white noise" if it is observed to have a flat spectrum over the range of frequencies [11]. Theoretically bandwidth of white noise is infinite. But the bandwidth of this noise is limited in practice by the mechanism of noise generation.

2.2. Color Noise

The color noise is generally characterized by its power spectral density. The noise of different color has different impact on signals. Power spectral density per unit of bandwidth is proportional to $1/f^\beta$. For white noise, $\beta=0$, for pink noise, $\beta=1$ and for brown noise, $\beta=2$.

In pink noise, the frequency spectrum is logarithmic space and it has equal power in bands that are proportionally wide. The power decreases by 3 db octave compared with white noise. Pink noise sounds more natural than white noise. It sounds like rushing water.

Brownian noise or Red noise may refer to any system where power spectral density decreases with increasing frequency. The name is after the corruption of Brownian motion. This is also known as ‘random walk’ noise [12].

Blue noise’s power spectral density increases 3 db per octave with increasing frequency over a finite frequency range. There are no concentrated spikes in energy. Retinal cells are arranged in a blue noise pattern which yields good visual resolution [13].

Violet noise’s power density increases 6 db per octave with increasing frequency over a finite frequency range. It is also known as differentiated white noise. Acoustic thermal noise of water has a violet spectrum [14].

Grey noise is random white noise over a certain frequency range .This is a contrast to standard white noise which has equal strength over a linear scale of frequencies.

3.ADAPTIVE FILTER FOR NOISE CANCELLATION

The principal of adaptive filtering is to obtain an optimum estimate of the noise and subtract it from the noisy signal. When the speech signal and noise contained in the primary input are uncorrelated and no crosstalk conditions are met, then adaptive noise cancelling techniques allow reduction of noise without information signal distortion. An adaptive filter works as the model that relates the primary input signals and adaptive filter output signal in real time in an iterative manner [15]. The concept of adaptive filter is shown in Fig. 1. It has a Finite Impulse Response (FIR) structure. For such structures, the impulse response is equal to the filter coefficients [16]. It is a nonlinear filter since its characteristics are not independent on the input signal and consequently the homogeneity conditions are not satisfied. If we freeze the filter parameters at a given instant of time, most adaptive filters are linear in the sense that their output signals are linear functions of their input signals [17]

The contaminated signal passes through the filter. The filter suppresses noises from the contaminated signal and this process does not require priory idea about the signal and noise. In adaptive noise cancellation, one channel is used as the input path of speech that is corrupted by the white or color noise and the other input is used as the reference white Gaussian noise. Color noise can be obtained by passing the white noise through a Chebyshev filter to get an output noise which is concentrated at the pass band region of the filter. The signal can be corrupted by white, color or both white and color noise.

The speech signal and noise are expressed as $s(n)$ and $x_1(n)$ respectively. The reference noise is expressed as $x(n)$ and output of the adaptive filter is denoted as $y(n)$ which is produced as close as possible of $x_1(n)$.The filter readjust itself in the continuous process. This continuous adjustment process minimizes the error between $x_1(n)$ and $y(n)$ [18].

Signal is uncorrelated with noise $x_1(n)$. The signal $s(n)$ and noise $x_1(n)$ combined to form the desired signal $d(n) = s(n) + x_1(n)$. Reference noise $x(n)$ is uncorrelated with the signal but correlated in some unknown way with noise $x_1(n)$ [19]. The difference of the output $y(n)$ and the primary input produces the system output.

$$e(n)=s(n)+x_1(n)-y(n) \quad (1)$$

where, $e(n)$, $s(n)$, $x_1(n)$ and $y(n)$ represent the error signal, the speech signal, the noise signal and the output of the adaptive filter respectively.

The adaptive filter automatically adjusts its own impulse responses. Thus proper algorithm is needed to readjust the changing conditions. And with the help of the algorithm it does the adaptive process to minimize the error signal. Our main perspective is to make the system output signal $e(n)$ which fits best in the least square sense to the signal $s(n)$. The goal is completed by feeding the output back to the filter and adjusting the filter through LMS algorithm. Finally this process minimizes the total system output signal [20].

Both side of Eq.1 is squared,

$$e^2 = s^2 + (x_1 - y)^2 + 2s(x_1 - y) \quad (2)$$

From the both sides of Eq.2, realizing that s is uncorrelated with x_1 and y yields [20],

$$E[e^2] = E[s^2] + E[(x_1 - y)^2] + 2E[s(x_1 - y)] = E[s^2] + E[(x_1 - y)^2] \quad (3)$$

The minimum mean square error (MSE) is [20],

$$\text{minimum}[e^2] = E[s^2] + \text{minimum}[(x_1 - y)^2] \quad (4)$$

When the filter is adjusted $E[(x_1 - y)^2]$ get minimized. The filter output y will be a best least square estimate of the primary noise. The minimized value of $E[(x_1 - y)^2]$ minimizes $[(e - s)^2]$. Since from Eq.1

$$(e - s) = (x_1 - y) \quad (5)$$

This results minimized output noise power. Finally the minimized output power maximizes the Signal to Noise Ratio (SNR).

4. LEAST MEAN SQUARE (LMS) ALGORITHM

The originators of LMS algorithm is Widrow and Hoff (1960). This algorithm adjusts the filter coefficients for minimizing the cost function and does not involve any matrix operation. It is also popular for its good tracking capabilities in stationary environment and does not suffer from numerical instability problem. Fig.2 shows the signal flow graph representation of LMS algorithm.

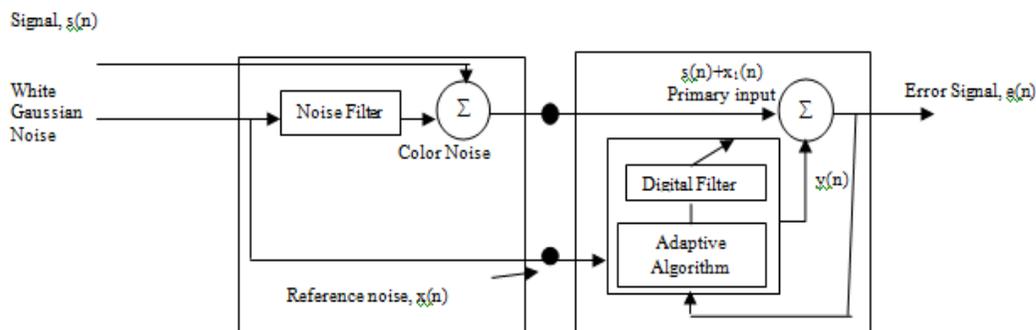


Figure 1: Adaptive noise cancellation system

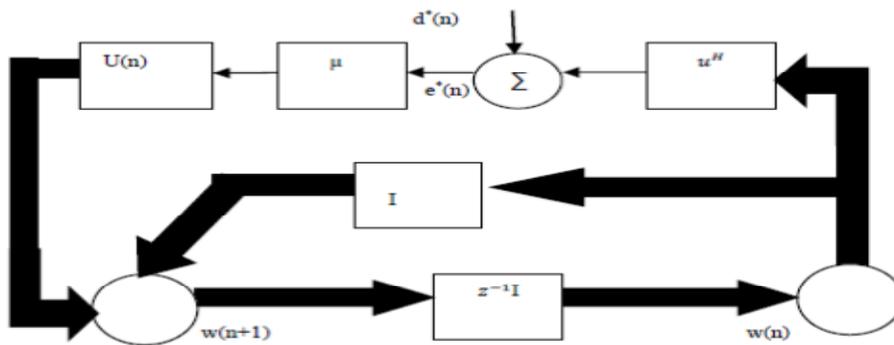


Figure 2: Signal flow graph representation of the LMS algorithm [21]

Two basic processes of LMS algorithm is given below:

- a. An LMS filtering process which involves computing the output of a linear filter in response to an input signal and generating an estimation error by comparing the output with a desired response.
- b. In accordance with the estimation error an adaptive process involve the automatic adjustment of the transvers filter's parameters [21].

The tap inputs for adaptive filter $u(n), u(n-1), \dots, u(n-M+1)$ form tap input vector $\mathbf{u}(n)$, where $M-1$ is the number of delay element. Correspondingly the tap weight $w_0(n), w_1(n), \dots, w(n)$ form the tap weight vector $\mathbf{w}_m(n)$. Using LMS algorithm the value computed for this vector represents an estimate whose expected value may come close to the Wiener solution \mathbf{w}_0 as the number of iteration approaches to infinity.

Desired response $d(n)$ of the filter, tap input vector $\mathbf{u}(n)$. Transvers filter produces an output as an estimate of the desired response $d(n)$.

$$\text{Estimation error, } e(n) = d(n) - u(n) \quad (6)$$

The basic LMS algorithm implementation procedure is given as follows.

1) Initially, set each weight $w_k(n)$ to an arbitrary fixed value, such as 0. For subsequent sampling instant $k=1, 2, \dots$ carry out steps (b) to (d) below:

2) Filter output [21] :

$$y(n) = \mathbf{w}^H(n) \mathbf{u}(n) \quad (7)$$

3) Error signal [21]:

$$e(n) = d(n) - y(n) \quad (8)$$

4) Tap weight adaptation [21]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) e^*(n) \quad (9)$$

where, μ , is the step size of the iteration

5.SIMULATIONS AND RESULTS

In this section, the performance analysis of adaptive noise cancelling system using LMS scheme will be reported. A comparison of the performances of adaptive noise canceller in case of white Gaussian noise situation and color noise situation has been made as well as the effect of number of iterations on the system's performance is investigated for both case. In addition, the effects of different parameters such as SNR, tap size on the performance of adaptive noise cancelling system are studied. The performance is measured in terms of MSE (Mean Square Error). All the experiments have been done using computer simulations implemented on MATLAB platform. A recorded speech clip as shown in Fig. 3 (duration: 4 seconds, number of samples: 32000 and sampling rate 8000 samples/second) has been used to evaluate the performance of the system.

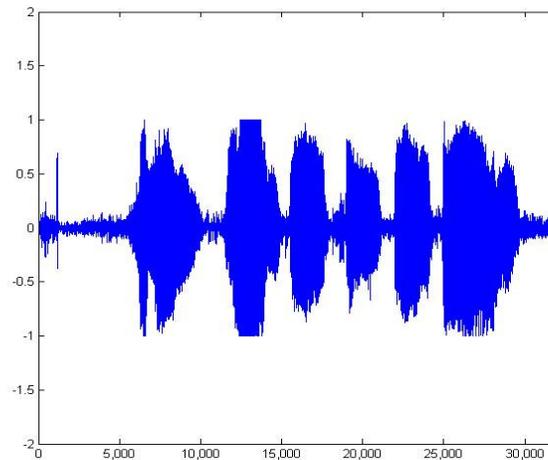


Figure 3: The original sound clip without any added noise

5.1.Noise Reduction using LMS Algorithm

In this section, we will consider two noise situation such as white Gaussian noise and colored noise to study the performance of adaptive noise canceller.

5.1.1.Case 1: Noise Reduction for White Gaussian Noise Situation

A white Gaussian noise (shown in Fig. 4) is purposely added to the original speech signal to produce contaminated speech signal (as seen in Fig. 5). The speech signal recovered by the adaptive noise cancelling system is shown in Fig. 6. A comparison between the original speech signal and the recovered speech signal is illustrated in Fig. 7. As seen from the figure that during convergence period there is a slight difference between the recovered signal and original signal. Once the system is converged, the recovered signal becomes identical to the original signal. It is observed from the Fig. 8 that approximately at 2500 iterations the system converged and after that the recovered signal became identical to the original signal.

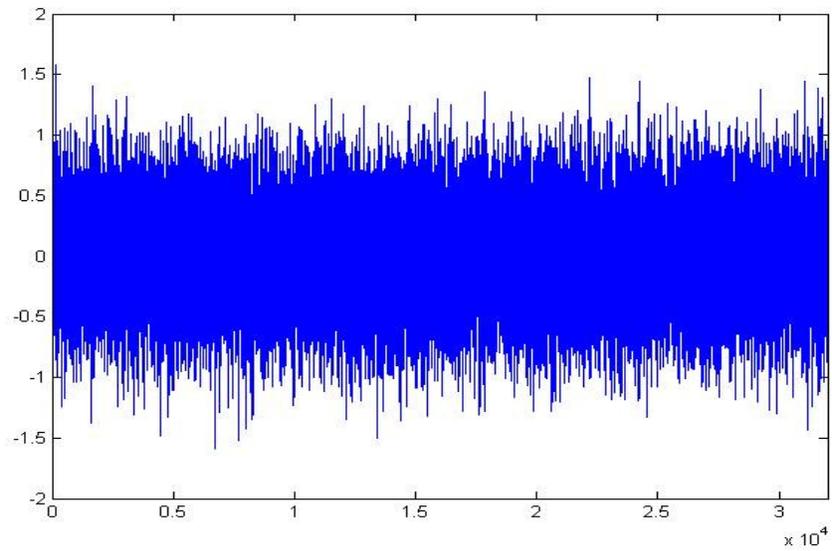


Figure 4: White Gaussian noise

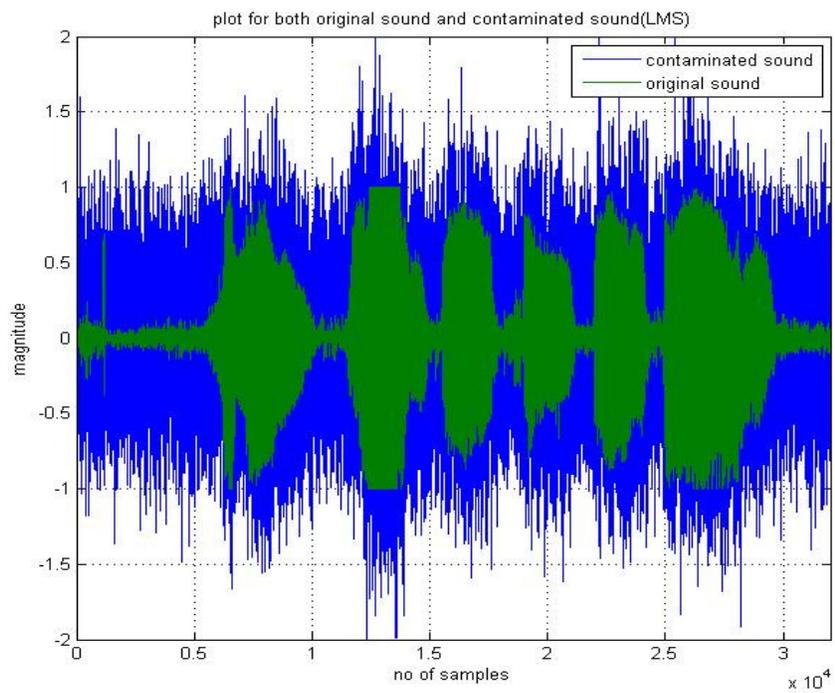


Figure 5: Noiseless voice signal (green) and voice signal contaminated with white Gaussian noise (blue)

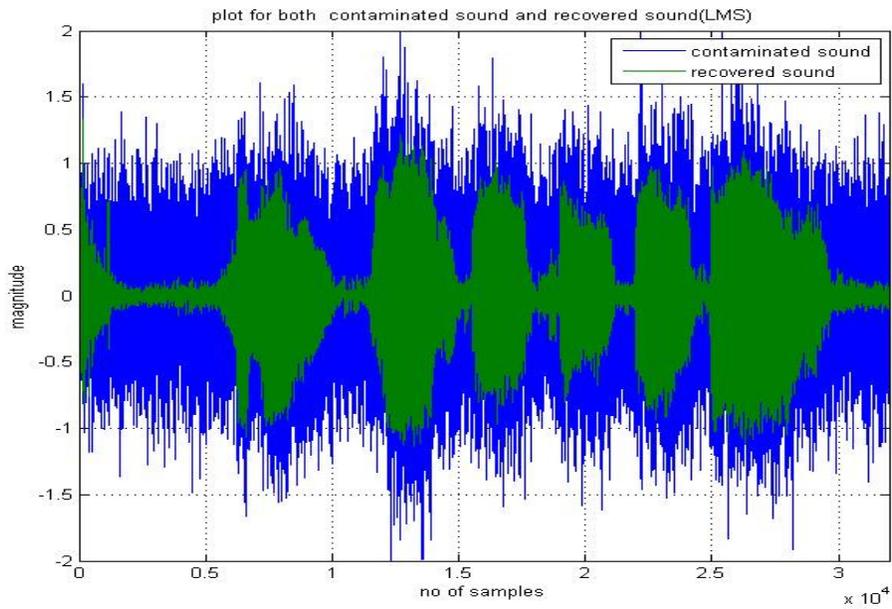


Figure 6: Recovered signal (green) and contaminated voice signal (blue)

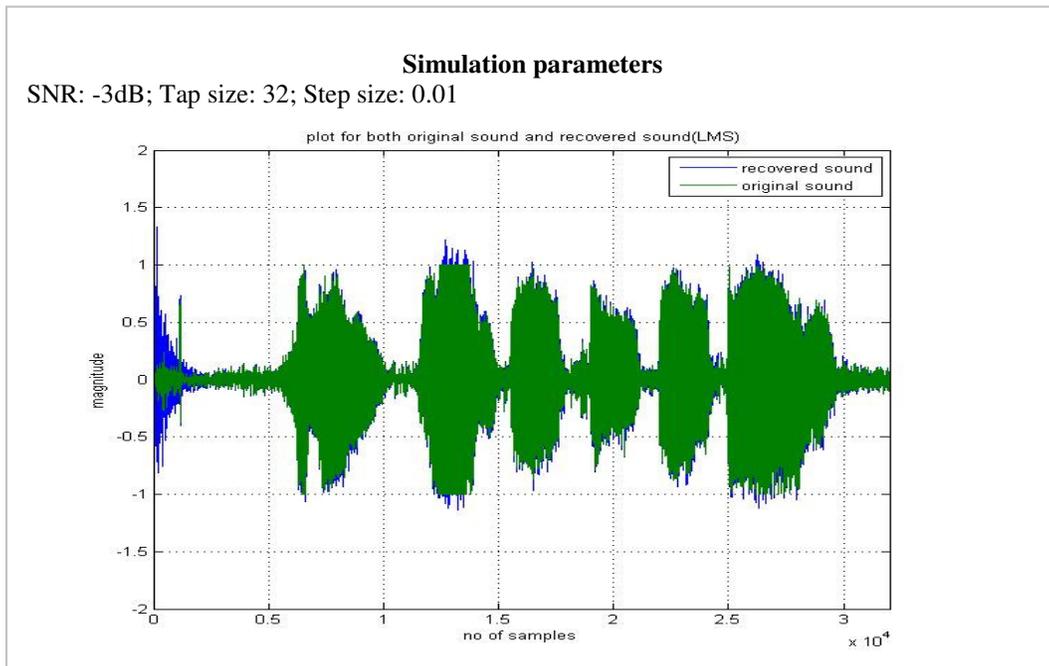


Figure 7: Plot of both original speech signal (green) and recovered signal (blue)

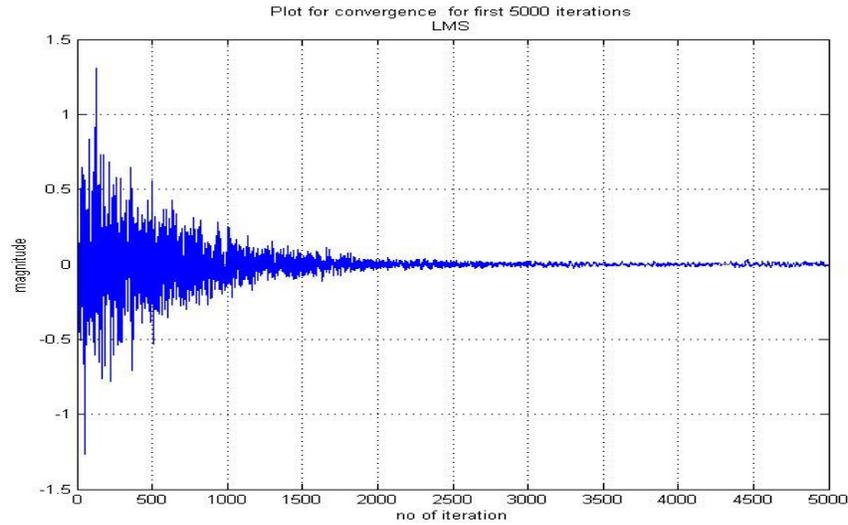


Figure 8: Plot of convergence for first 5000 iterations for reducing white Gaussian noise

5.1.2 Case 2: Noise Reduction for Color Noise Situation

The white Gaussian noise shown in Fig. 4 is passed through the Chebyshev filter to get colored noise which is then added to the original speech signal to produce contaminated speech signal (as seen in Fig. 10). The frequency response of Chebyshev filter is shown in Fig. 9. The recovered speech signal is shown in Fig. 11. A comparison between the original speech signal and the recovered speech signal is made in Fig. 12. Similar to Case 1, it is seen from the figure that during convergence period there is a slight difference between the recovered signal and original signal. Once the system is converged, the recovered signal becomes similar to the original signal. It is observed from the Fig. 13 that approximately at 2500 iterations the system converged.

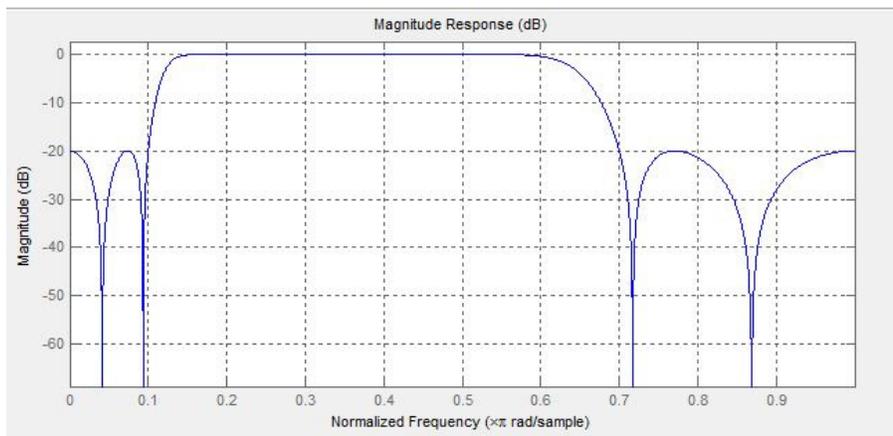


Figure 9: Frequency response of Chebyshev filter

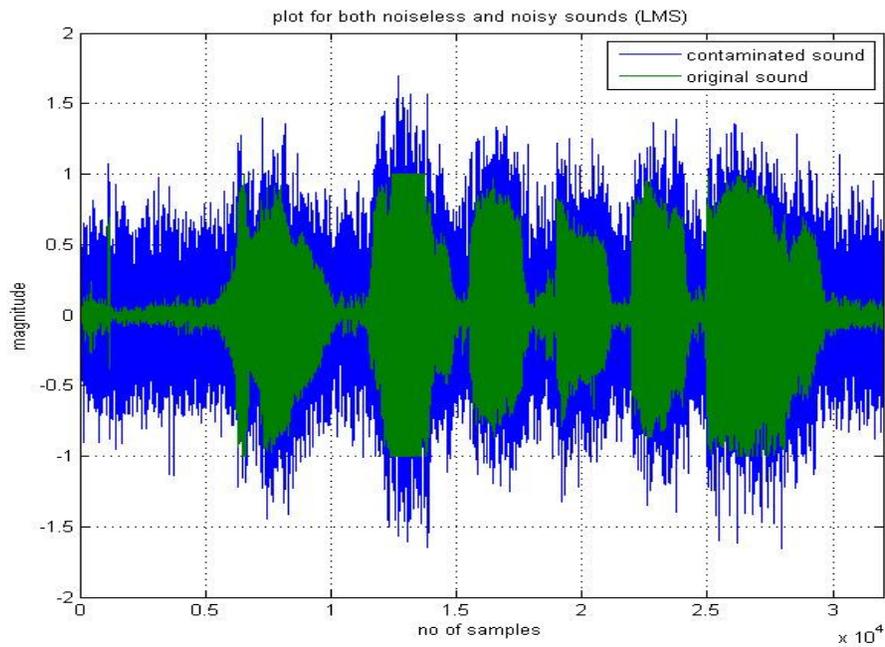


Figure 10: Noiseless voice signal (green) and voice signal contaminated by color noise (blue)

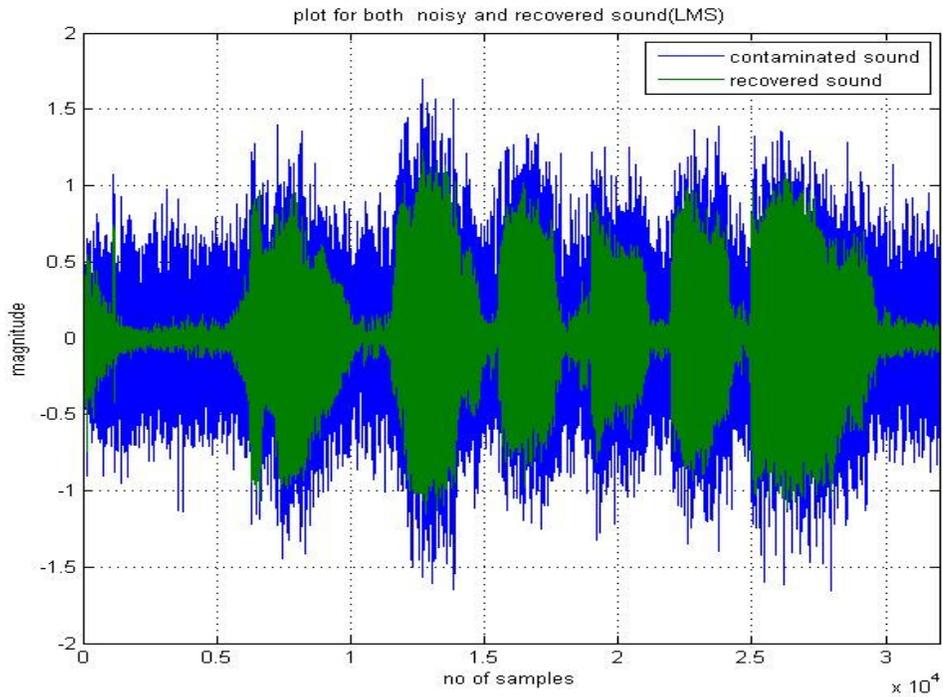


Figure 11: Plot of contaminated voice signal (blue) and recovered voice (green)

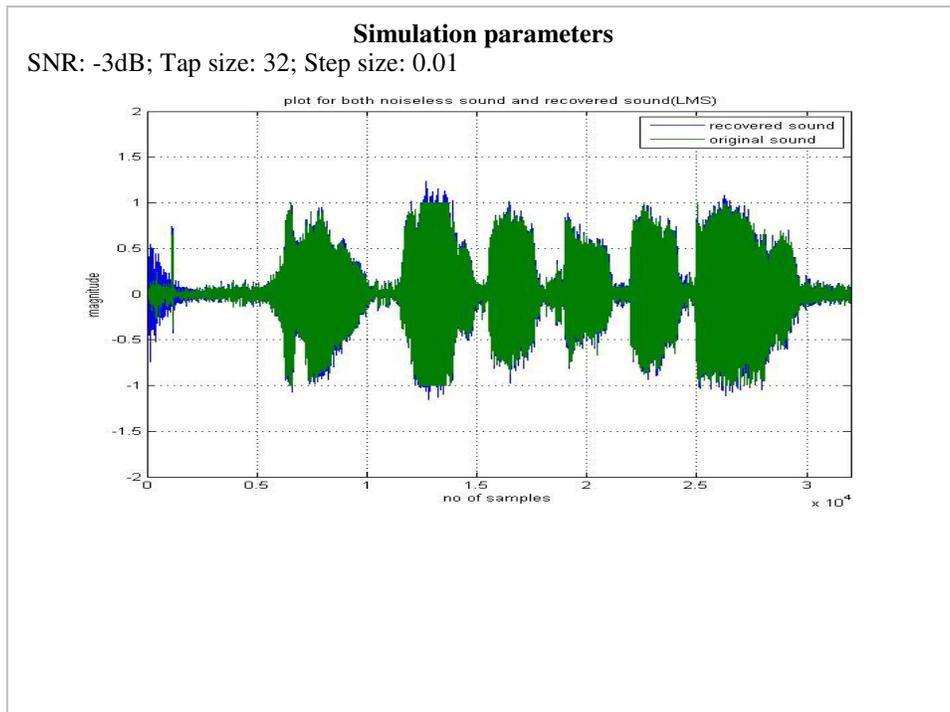


Figure 12: Plot of both original signal (green) and recovered signal (blue)

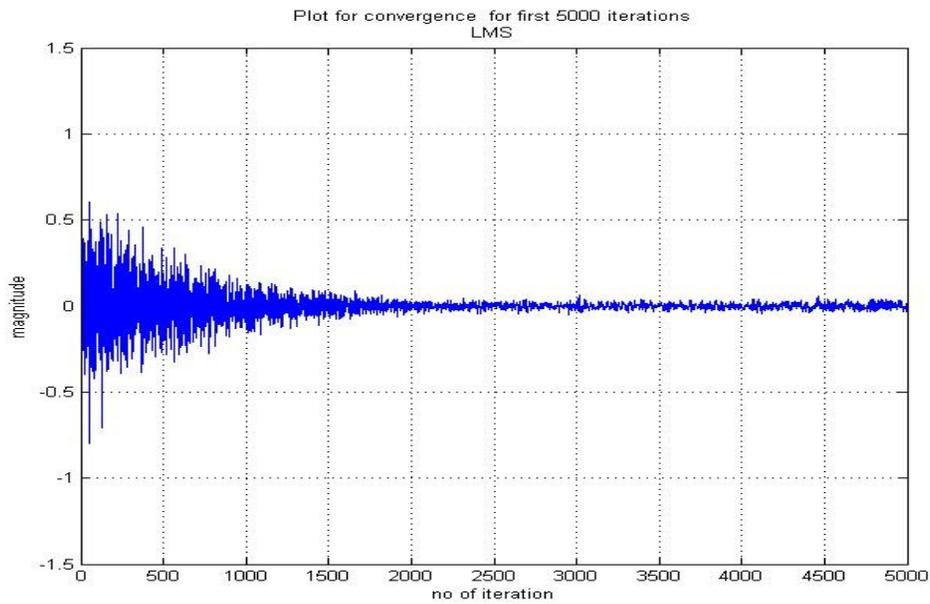


Figure 13: Plot of convergence for first 5000 iterations for reducing color noise

5.2. Mean Square Error (MSE) versus Number of Iterations Analysis

A comparison between Mean Square Error (MSE) versus number of iteration curves for white Gaussian noise situation and the color noise situation is made in this section. The MSE versus number of iterations graph shown in Fig. 14 illustrates that for white Gaussian noise, the peak of the MSE is greater than 0.14, whereas for color noise case it is greater than 0.06 as seen in Fig. 15. In both case, after some initial fluctuations, the MSE declines with increasing number of iterations and at each point the MSE for white noise is greater than that for color noise. Therefore, with making a comparison between Fig. 14 and Fig. 15 it can be concluded that for color noise case, noise reduction is better and speed of convergence is faster than those for white noise case.

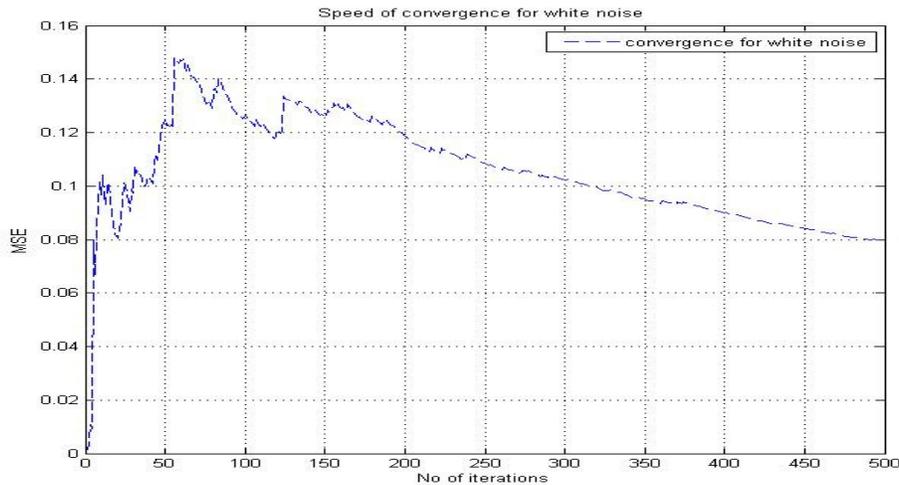


Figure 14: MSE versus number of iterations for white noise situation

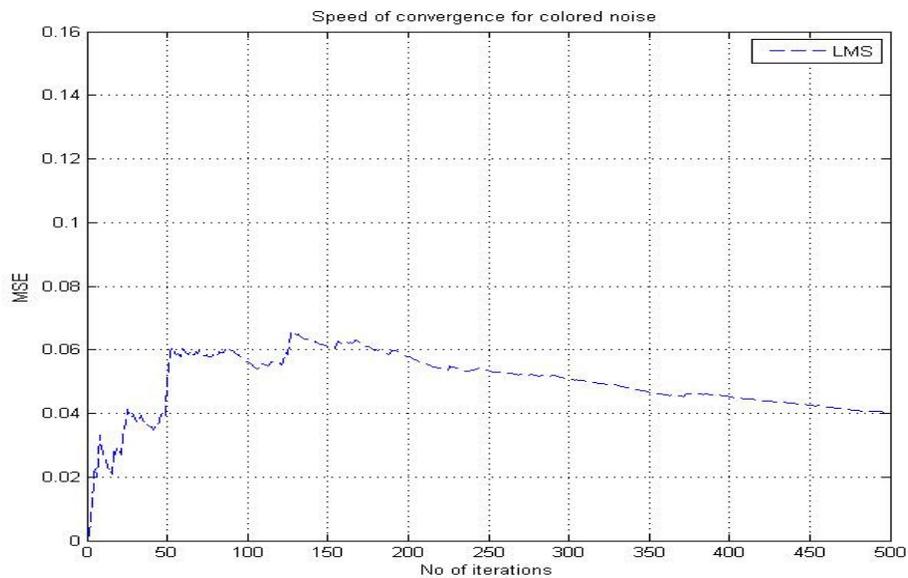


Fig. 15: MSE versus number of iteration for colored noise situation

5.3. Effects of Signal to Noise Ratio (SNR) on Mean Square Error (MSE)

The effects of SNR on the mean square error is reported in this section. The results obtained in case of white Gaussian noise and color noise are tabulated in Table 1 and Table 2 and the graphical representations of the results are provided in Fig. 16 and Fig. 17 respectively. It can be observed that for both case MSE decreases with increasing SNR with relatively faster decrease of MSE in case of color noise compared with white noise. It is also noticeable that at high SNR we have obtained better MSE in color noise case.

Table 1: Effects of SNR on MSE for white noise

| Simulation parameters | | | |
|------------------------------|---------|---------|---------|
| Step size: 0.01 | | | |
| Tap size: 32 | | | |
| SNR(dB) | MSE | SNR(dB) | MSE |
| -10 | .013055 | 3 | .002060 |
| -9 | .010333 | 4 | .001967 |
| -8 | .008289 | 5 | .001894 |
| -7 | .006745 | 6 | .001834 |
| -6 | .005573 | 7 | .001786 |
| -5 | .004680 | 8 | .001746 |
| -4 | .003998 | 9 | .001712 |
| -3 | .003474 | 10 | .001677 |
| -2 | .003070 | 11 | .001635 |
| -1 | .002757 | 12 | .001577 |
| 0 | .002514 | 13 | .001499 |
| 1 | .002324 | 14 | .001397 |
| 2 | .002176 | 15 | .001277 |

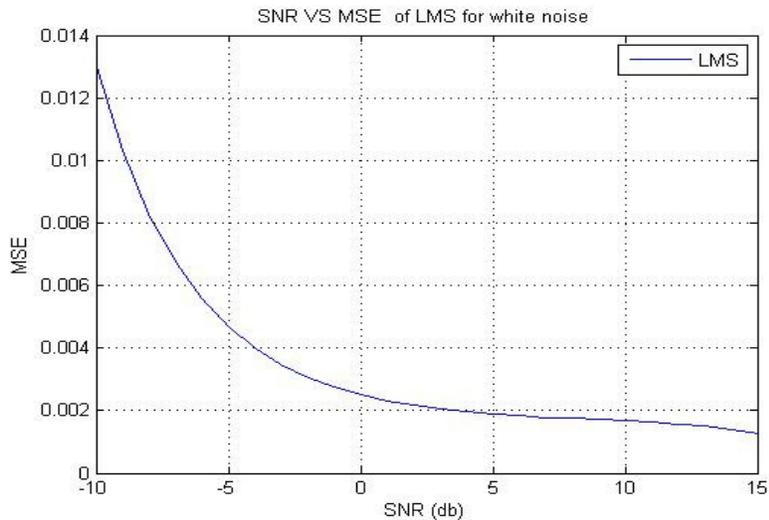


Figure 16: SNR versus MSE for white noise

Table 2: Effects of SNR on MSE for color noise

| Simulation parameters | | | |
|------------------------------|---------|---------|---------|
| Step size: 0.01 | | | |
| Tap size: 32 | | | |
| SNR(dB) | MSE | SNR(dB) | MSE |
| -10 | .013115 | 3 | .001330 |
| -9 | .010198 | 4 | .001228 |
| -8 | .008008 | 5 | .001145 |
| -7 | .006354 | 6 | .001076 |
| -6 | .005099 | 7 | .001017 |
| -5 | .004143 | 8 | .000969 |
| -4 | .003412 | 9 | .000928 |
| -3 | .002851 | 10 | .000892 |
| -2 | .002417 | 11 | .000856 |
| -1 | .002082 | 12 | .000817 |
| 0 | .001820 | 13 | .000771 |
| 1 | .001617 | 14 | .000716 |
| 2 | .001457 | 15 | .000653 |

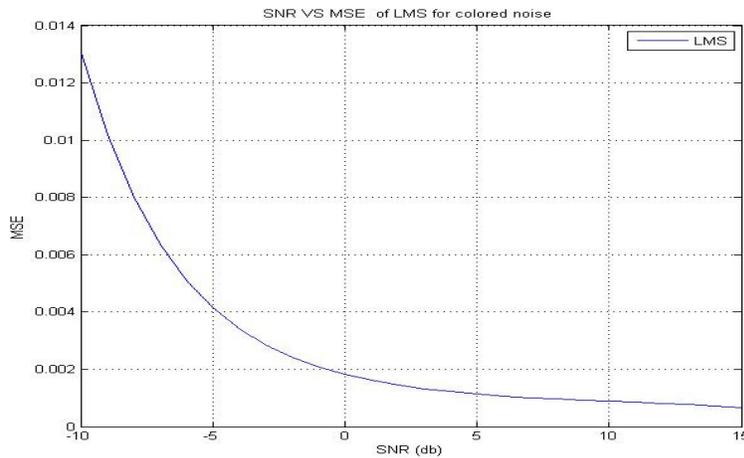


Figure 17: SNR versus MSE for color noise

5.4. Effects of Tap Size on Mean Square Error (MSE)

The effects of tap size on MSE are tabulated in Table 3 (for white noise) and Table 4 (for color noise) and the graphical representations of the results are shown in Fig. 18 (for white noise) and Fig. 19 (for color noise). Fig. 18 illustrates that for white noise, the MSE linearly increases with increasing tap size. For color noise as seen in Fig. 19, there is some initial randomness of the values of MSE till a particular tap length (25) after which the MSE increases almost linearly with increasing number of filter coefficients.

Table 3: Effects of number of taps on MSE for white noise

| Simulation parameters | | | |
|------------------------------|---------|-----|---------|
| SNR: -3dB | | | |
| Step size: 0.01 | | | |
| TAP | MSE | TAP | MSE |
| 5 | .001850 | 20 | .002726 |
| 6 | .001893 | 21 | .002788 |
| 7 | .001930 | 22 | .002884 |
| 8 | .001974 | 23 | .002950 |
| 9 | .002024 | 24 | .003017 |
| 10 | .002088 | 25 | .003069 |
| 11 | .002158 | 26 | .003138 |
| 12 | .002230 | 27 | .003203 |
| 13 | .002277 | 28 | .003271 |
| 14 | .002338 | 29 | .003318 |
| 15 | .002385 | 30 | .003374 |
| 16 | .002444 | 31 | .003427 |
| 17 | .002514 | 32 | .003474 |
| 18 | .002604 | 33 | .003513 |
| 19 | .002674 | 34 | .003555 |

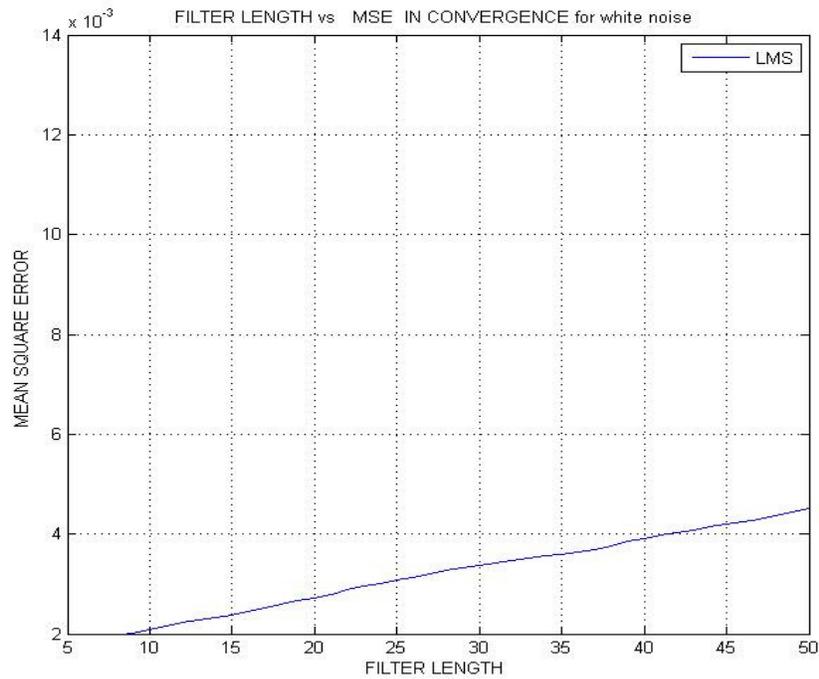


Figure 18: TAP size versus MSE for white noise case

Table 4: Effects of number of taps on MSE for color noise

| Simulation parameters | | | |
|------------------------------|---------|-----|---------|
| SNR: -3dB | | | |
| Step size: 0.01 | | | |
| TAP | MSE | TAP | MSE |
| 5 | .012439 | 20 | .002809 |
| 6 | .010210 | 21 | .002831 |
| 7 | .010172 | 22 | .002864 |
| 8 | .009843 | 23 | .002856 |
| 9 | .004618 | 24 | .002728 |
| 10 | .004632 | 25 | .002652 |
| 11 | .003724 | 26 | .002661 |
| 12 | .003668 | 27 | .002673 |
| 13 | .003403 | 28 | .002732 |
| 14 | .003447 | 29 | .002777 |
| 15 | .003104 | 30 | .002827 |
| 16 | .002730 | 31 | .002853 |
| 17 | .002802 | 32 | .002851 |
| 18 | .002809 | 33 | .002852 |
| 19 | .002835 | 34 | .002861 |

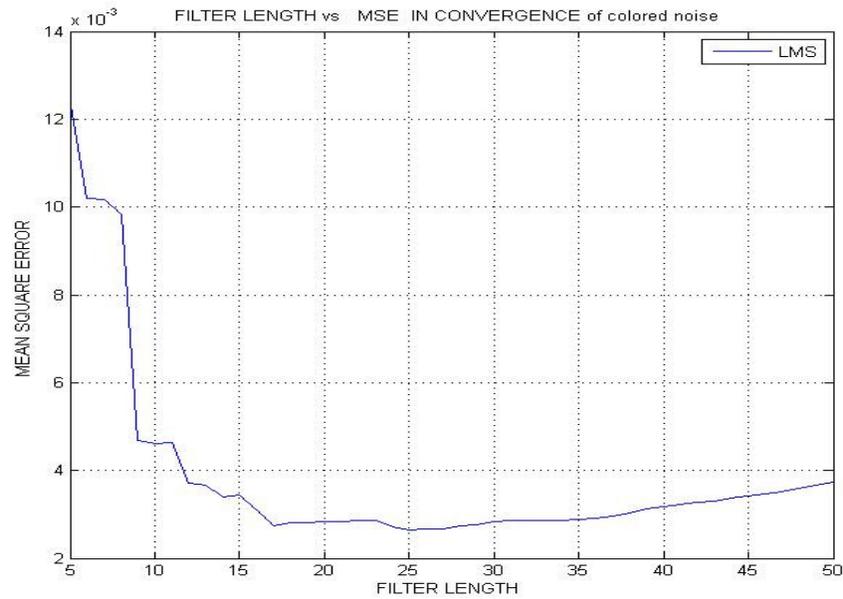


Figure 19: TAP size versus MSE for color noise case

6.CONCLUSION

In this paper, we have considered white Gaussian noise and color noise to study the performance of adaptive noise cancelling system. A comparison between the performances of Case 1 (for white noise) and Case 2 (for color noise) is made as well as the effects of number of iterations, SNR and tap size on the MSE are studied for both case. It is seen that the reduction of noise is better and speed of convergence is faster for color noise as compared with white noise situation. It has been found that MSE increased almost linearly with the filter length in case of white noise, whereas for color noise, some initial randomness of the values of MSE is observed until a particular tap length (25) after which the MSE increases almost linearly with increasing number of filter coefficients. Our experiments to find the possible relationship between SNR and MSE reveals that for both case, MSE decreases with increasing SNR with relatively faster decrease of MSE in case of color noise compared with white noise. It is noticeable that at high SNR, better MSE is obtained for Case 2. Our future plans include to study other adaptive algorithms and analyse their effects on the performance of noise cancelling system.

REFERENCES

- [1] E. C. Ifeachor and B. W. Jervis, Digital Signal Processing, Addison-Wesley Publishing Company, 1993.
- [2] S. Haykin, Adaptive Filter Theory, 3RD edition, Prentice-Hall International Inc, 1996.
- [3] F. Afroz, A. Huq, F. Ahmed and K. Sandrasegaran. "Performance Analysis of Adaptive Noise Canceller Employing NLMS Algorithm," International Journal of Wireless and Mobile Networks, vol.7. no. 2, pp. 45-58, April 2015.
- [4] B. Widrow and M.E. Hoff, "Adaptive switching circuits," Proceedings of WESCON Convention Record, part 4, pp.96-104, 1960.
- [5] R. W. Harris, D.M. Chabries, and F. A. Bishop, "A variable step (VS) adaptive filter algorithm," IEEE Trans. Acoustics, Speech, Signal Processing, vol. ASSP-34, pp. 309-316, April 1986.
- [6] A. Kanemasa and K. Niwa, "An adaptive-step sign algorithm for fast convergence of a data echo canceller," IEEE Trans. Communications, vol. COM-35, NO. 10, pp. 1102-1 106, October 1987.
- [7] W. B. Mikhael et al., "Adaptive filters with individual adaptation of parameters," IEEE Trans. Circuits and Systems, vol. CAS-33, pp. 677-685, July 1986.
- [8] V. J. Mathews and Z. Xie, "A stochastic gradient adaptive filter with gradient adaptive step size," IEEE Trans. Signal Processing, vol. 41, pp. 2075-2087, June 1993.
- [9] M. H. Puder, and G.U. Schmidt, "Step-size control for acoustic echo cancellation filter-an overview," Signal Processing, pp. 1697-1719, September 2000.
- [10] S. Koike, "A class of adaptive step-size control algorithms for adaptive filters," IEEE Trans. Signal Processing, vol. 50, pp. 13 15- 1326, June 2002.
- [11] B. Carter and R. Mancini, Op Amps for everyone, 3rd Edition, Texas Instruments, 2009.
- [12] D.L. Rudnick and R.E. Davis, "Red noise and regime shifts," Deep-Sea Research Part I, vol.50(6), pp. 691-699, 2003.
- [13] D.P. Mitchell, "Generating Antialiased Images at Low Sampling Densities," ACM SIGGRAPH Computer Graphics, vol.21(4), pp. 65-72, 1987.
- [14] C. Roads, Composing Electronic Music: A New Aesthetic, Oxford University Press, May 2015.
- [15] S.C. Douglas, "Introduction to Adaptive Filters" in Digital Signal Processing Handbook, Ed. Vijay K. Madisetti and Douglas B. Williams, Boca Raton: CRC Press LLC, 1999.
- [16] I. Ahmad, F. Ansari and U.K. Dey, "Cancellation of motion artifact noise and power line interference in ECG using adaptive filters," International Journal of Electronics Signals and Systems (IJESS), Vol-3, Iss-2, pp 56-58, 2013.

- [17] V. Anand, S. Shah and S. Kumar, "Intelligent Adaptive Filtering For Noise Cancellation," International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol. 2, Issue 5, pp. 2029-2039, May 2013.
- [18] S. Yoon, S. Nat, S. Park, Y. Eom and S. Yoo, "Advanced Sound Capturing Method with Adaptive Noise Reduction System for Broadcasting Multi copters," IEEE International Conference on Consumer Electronics, pp. 26-29, 2015.
- [19] Siddappaji and K.L. Sudha, "Performance Analysis of New Time Varying LMS (NTVLMS) Adaptive Filtering Algorithm in Noise Cancellation System," International Conference on Communication, Information & Computing Technology (ICCICT), pp. 1-6, 2015.
- [20] G. Singh, K. Savia, S. Yadav and V. Purwar, "Design of adaptive noise canceller using LMS algorithm," International Journal of Advanced Technology & Engineering Research (IJATER), Vol.3, Issue 3, May 2013.
- [21] S. Haykin, "Least-Mean-Square Adaptive Filters," in Adaptive Filter Theory, 4th Edition, Singapore, Pearson Education, pp. 231-234, 2002.